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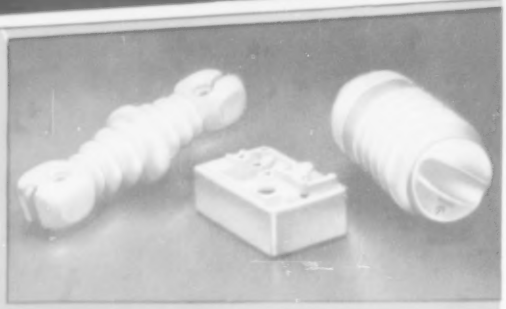

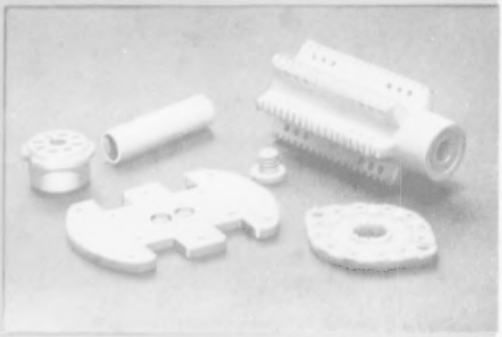

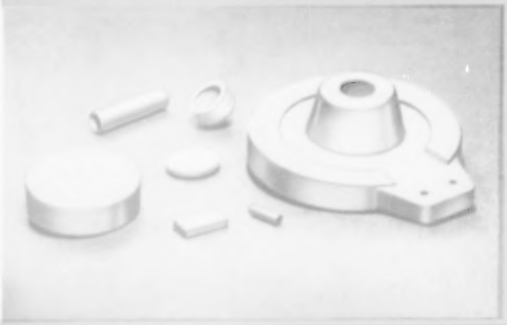

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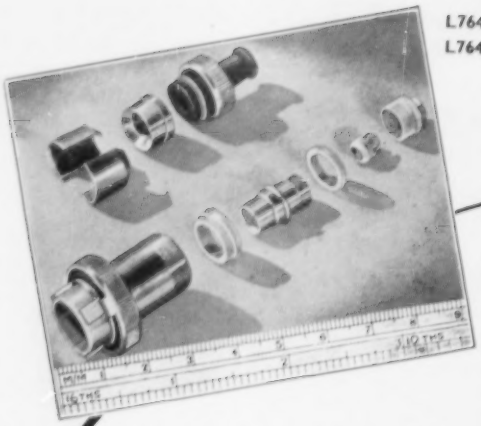

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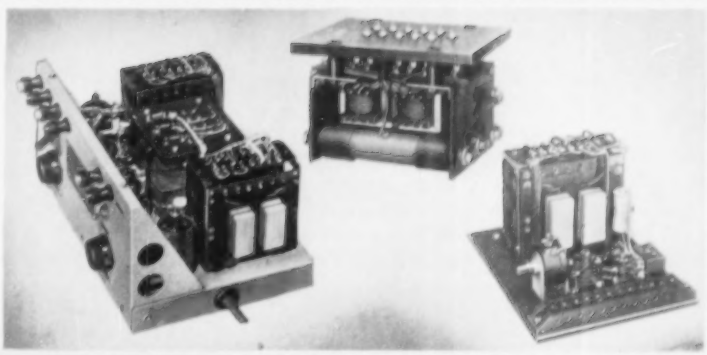

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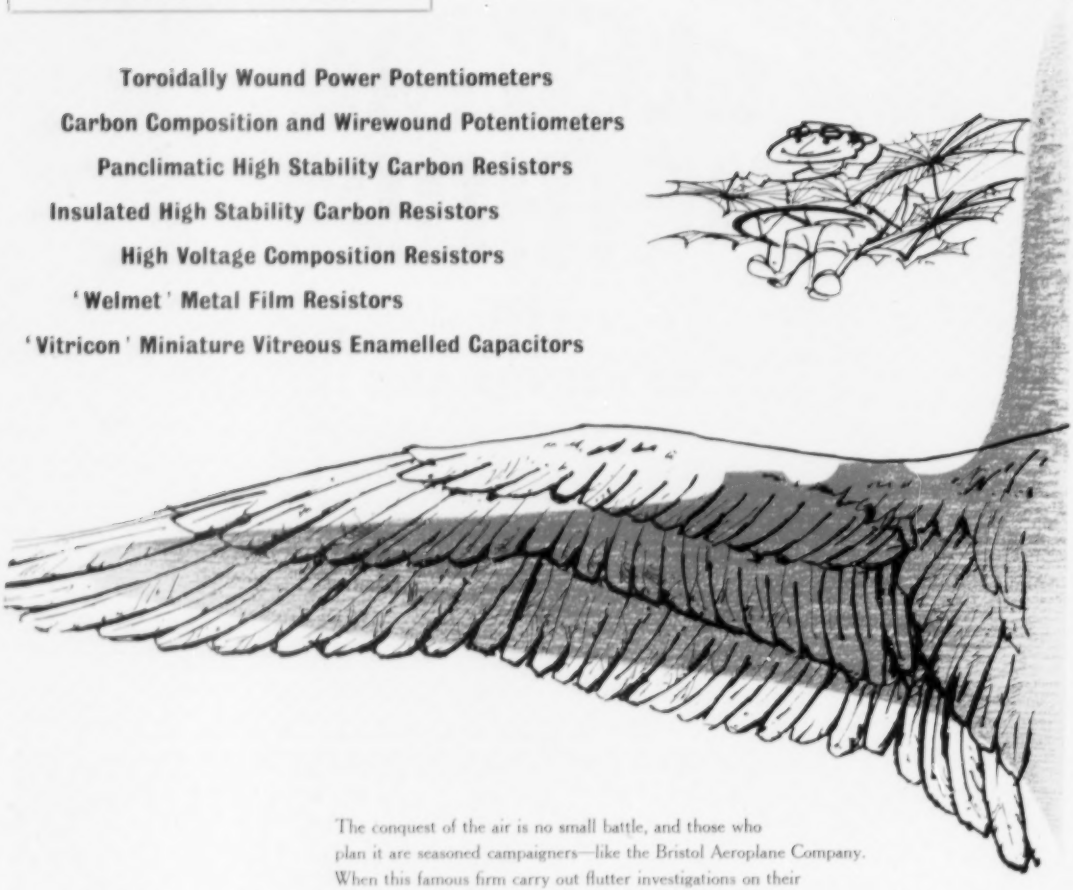

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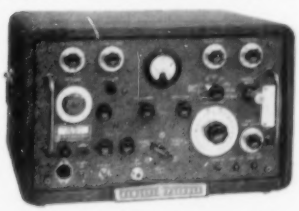

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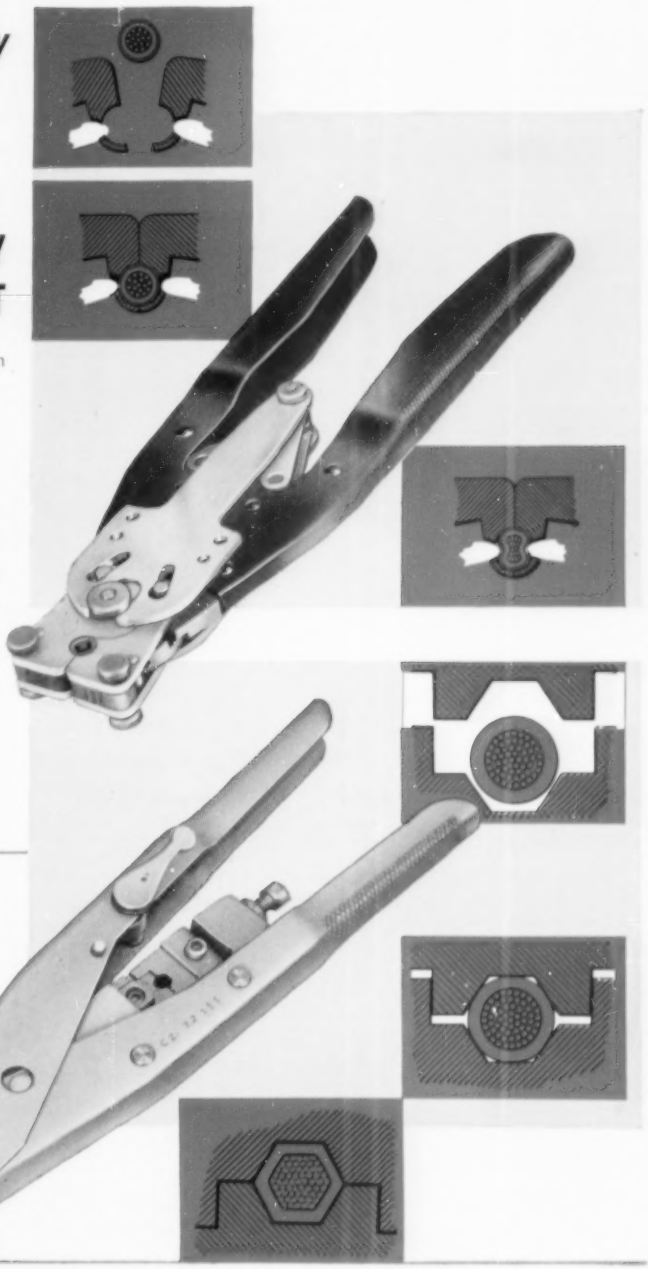

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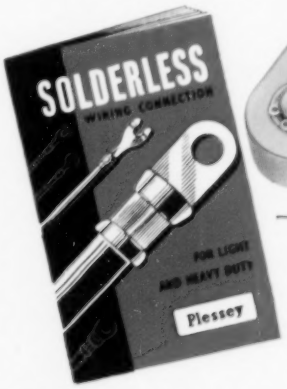

Plessey publication 753/1 contains extensive details of these tools and their application. A copy will be sent to manufacturers free, on request.

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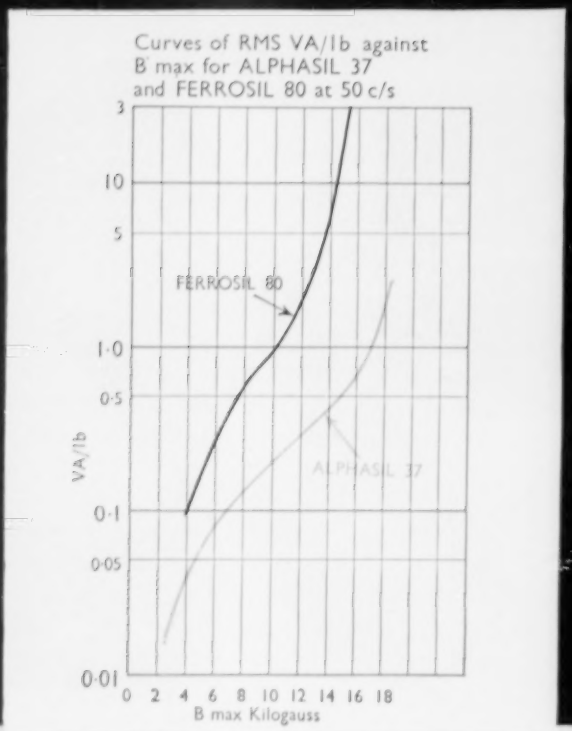

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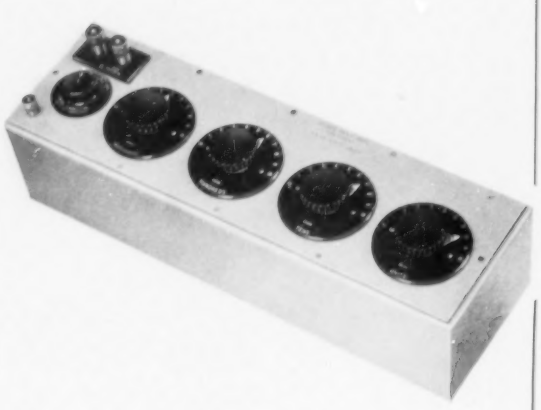

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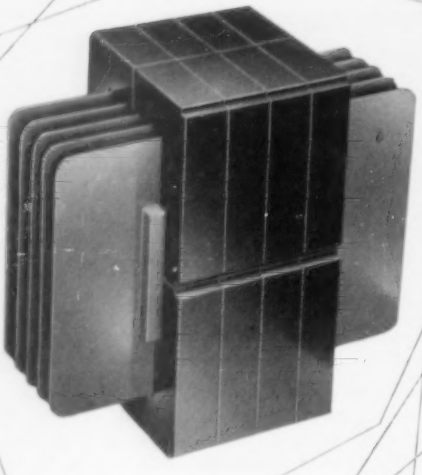

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Vol. 33 MAY 1956 No. 5

The Bifilar-T Trap

IN last month's Editorial, we discussed a new rejector circuit which is known as the bifilar-T trap Fig. 1(a) and we derived an equivalent circuit for it Fig. 1(b). The equivalence requires $L' = 2L_1$, $C' = C_1/2$, $R' = 2R$, $L_e = 4LL_1/(L - L_1)$, $C_e = C_1/4$ when R is equal to the dynamic resistance of L_1C_1.

When used as a coupling element between valves, the input and output terminals of Fig. 1(a) will inevitably be shunted by resistance and capacitance and, if desired, they can be shunted by inductance also. The equivalent circuit of Fig. 1(b) still applies, but the values of L', C' and R' become changed to take into account these external elements.

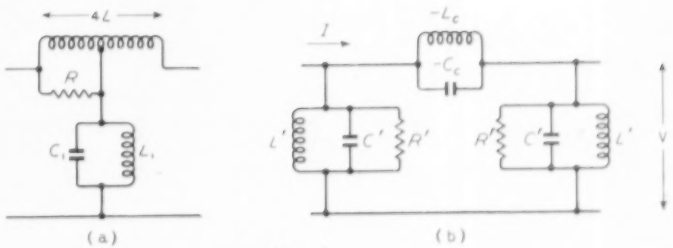

Fig. 1.

For simplicity, we shall assume that the two shunt arms are alike, which means that the same values of external elements must be added at each end. The transfer admittance is then

$$Y_T = I/V = Y'(2 + Y'Z_e)$$

where Y' is the admittance of a shunt arm and Z_e the impedance of the series arm.

It is a straightforward matter to expand this to the form

$$|Y_TR'|^2 = [1 + (QF - QH)^2]\left[\left\{2 - \frac{L_e}{L'} + \frac{L_e}{L'}\cdot\frac{QH}{QF}\right\}^2 + \left(\frac{L_e}{L'}\cdot\frac{1}{QF}\right)^2\right]$$

where $Q = R'/\omega_eL'$; $F = 2\Delta f/f_e$; $H = 2\delta f/f_e$;
and $f_e = \omega_e/2\pi$ = resonance frequency of the L_eC_e arm
Δf = frequency difference from f_e
δf = difference of resonance frequency of an $L'C'$ arm from f_e.

In deriving this equation, the common assumptions that $\Delta f/f_e$ amd $\delta f/f_e$ can be neglected in comparison with unity have been made. The error due to this probably does not exceed 10%.

Taking QF as the frequency variable, the performance depends upon two factors, the relative detuning of the series and shunt arms QH and the effective coupling between the shunt arms L_e/L'. By taking a series of values of each, the performance can be expressed in the form of families of curves relating Y_TR' to QF. It is easy but tedious to compute such families. We have not attempted this, but we have calculated one curve for arbitrarily selected values ($QH = -1$ and $L_e/L' = 2{\cdot}5$) and this is shown in Fig. 2. As will appear later, we made a lucky choice with these values, for they are of the right order for a television i.f. amplifier.

It is worth noting that there is a minimum possible value of 2 for L_e/L'. The ratio tends to this when $L \gg L_1$ and there is no external shunt inductance.

When the response is zero dB in Fig. 2, the value of Y_TR' is 2, which means that the maximum transfer impedance is $R'/2$. This is,

of course, the normal value for a pair of circuits with optimum coupling.

As an intervalve coupling in a television i.f. amplifier, we might want $f_c = 38$ Mc/s and a response of -2 dB at 34·5 Mc/s and 37·5 Mc/s. For the latter, $\Delta f = -0{\cdot}5$ Mc/s and from Fig. 2 $QF = -1{\cdot}33$ so

$$Q = \frac{-1{\cdot}33 \times 38}{-0{\cdot}5 \times 2} = 50{\cdot}7$$

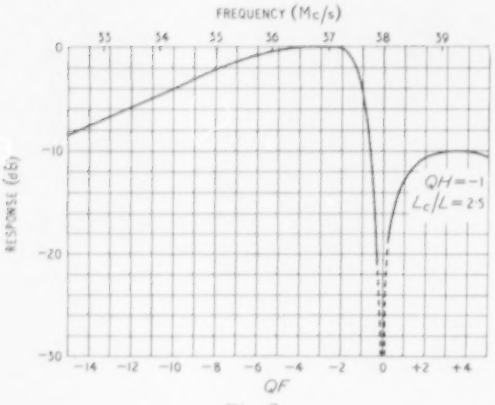

Fig. 2.

At 34·5 Mc/s, $\Delta f = -3{\cdot}5$ Mc/s and so

$QF = -3{\cdot}5 \times 50{\cdot}7 \times 2/38 = -9{\cdot}5$

and the curve shows the response to be $-3{\cdot}65$ dB. The required conditions are thus not met but the difference is not very great; we judge that a slightly smaller value of L_c/L' would enable the specification to be met.

Since $QH = -1$ we have

$$\delta f = \frac{-1 \times 38}{2 \times 50{\cdot}7} = -0{\cdot}375 \text{ Mc/s.}$$

The shunt arms are thus tuned between the upper limit of the pass-band and the rejection frequency.

A frequency scale has been added in Fig. 2 for the particular conditions just discussed, so that the characteristics can more readily be seen.

It is important to determine the values of the elements in the real circuit of Fig. 1(a). A typical modern valve will have an input resistance of the order of 20 kΩ and a capacitance of some 10 pF while the output capacitance may be 5 pF.

Let us assume, therefore, that the circuit of Fig. 1(a) is to be terminated at each end by 10 pF shunted by 20 kΩ. These values are to be absorbed in C' and R' of Fig. 1(b). From the relations given earlier $L/L_1 = 5$, since $L_c/L' = 2{\cdot}5$.

Also $L_cC_c = 1/\omega_c^2$ and $L'C' = 1/\omega_c^2(1 - \delta f/f_c)^2$, so $L_cC_c/L'C' = 0{\cdot}98$ and $C'/C_c = 2{\cdot}55$.

Now $C_c = C_1/4$ and $C' = C_s + C_1/2$ where C_s is the external capacitance, hence

$$\frac{C_1}{C_s} = \frac{4}{C'/C_c - 2} = \frac{4}{0{\cdot}55} = 7{\cdot}3$$

with $C_s = 10$ pF, $C_1 = 73$ pF; and $C' = 46{\cdot}5$ pF, $C_c = 18{\cdot}25$ pF.

From the frequency relation, $L_c = 0{\cdot}97$ μH, where $L' = 0{\cdot}388$ μH, $L_1 = 0{\cdot}194$ μH and $L = 0{\cdot}97$ μH. With $Q = 50{\cdot}7$

$$R' = \omega_c L' Q = 4{\cdot}65 \text{ kΩ}$$

This includes the effect of the 20-kΩ shunt. Without it, the resistance becomes 6·04 kΩ and so R and R_1 must be 3·025 kΩ. This is the value of the dynamic resistance of L_1C_1 and so the required Q for this circuit is $3{\cdot}025 \times 10^3/\omega_c L_1 = 66$. This is readily possible.

The final circuit with values thus takes the form shown in Fig. 3. The L_1/C_1 ratio is actually rather low but, although we have not considered this, it can obviously be changed at will by connecting the bifilar coil to a tap on the coil L_1. In practice, one would use about one-third of the capacitance and three times the inductance and tap down the circuit to keep the impedance unchanged.

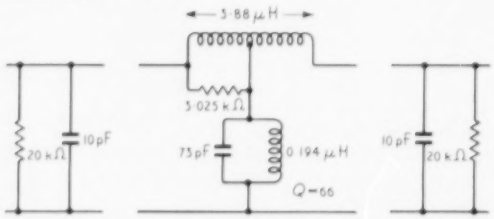

Fig. 3.

The transfer resistance at mid-band is $4{\cdot}65/2 = 2{\cdot}325$ kΩ so that, with suitable valves, stage gains of the order of 20 times are possible.

The values which we have deduced here cannot be considered as precise ones, for in all this work we have assumed negligible losses, self-capacitance and leakage inductance in the bifilar coil. In practice, they may not be negligible and they may well require changes in the values of other components. Nevertheless, the values should afford a good starting point for experimental work.

Reference to Fig. 4(a) of the April Editorial shows that, apart from the external capacitance, the frequency of $L'C'$ depends only on L_1C_1, whereas the rejection frequency depends on L, L_1 and C_1. Plainly then, L_1C_1 should be tuned to $f_c - \delta f$, for the required shape of pass-band, and L should be adjustable afterwards for maximum attenuation at f_c.

Inspection of the figures obtained during the

calculation of the curve of Fig. 2 shows that the falling off in response at large negative values of QF is due to the first bracket of the equation, which actually represents the response of a single tuned circuit. The second bracket by itself gives an increasing response.

It is plain, therefore, that tuning the shunt arms to a lower frequency would improve the low-frequency response. This would, however, be at the expense of the high-frequency end. Increasing L_c/L' tends to narrow the bandwidth.

Apart from varying QH and L_c/L', there is the possibility of staggering the tuning of the shunt arms by using unequal capacitance and/or inductance shunts external to the basic network. Theoretical investigation of this becomes difficult, but it is a very practical way of modifying the performance. W.T.C.

GROUP-DELAY MEASUREMENTS

Use of Wobbulator and C.R. Display

By C. J. Heuvelman and A. van Weel

(Philips Research Laboratories, N. V. Philips' Gloeilampenfabrieken, Eindhoven, Holland)

SUMMARY.—A description is given of a simple group-delay meter which, in combination with any conventional wobbulator generator, gives the group-delay characteristic directly on an oscilloscope. An automatic-gain-control circuit, necessary to maintain a constant level at the output of the network under test, enables the tracing of the amplitude characteristic on a second oscilloscope at the same time. Calibration of amplitude and group-delay scales is possible for any oscilloscope used. A sensitivity of 1 milli-microsecond can be achieved. The frequency range is 20–45 Mc/s.

Introduction

In recent years, the measurement of group-delay characteristics of electric networks has been the subject of many investigations and a number of different measuring devices has been described in the literature[1–8]. In general, these group-delay measuring equipments are rather intricate and often not too easy to handle. As a consequence, their use has mainly been restricted to research laboratories or to the development of professional transmitting equipments for coaxial cables, radio-relay systems, etc.

However, the problems posed by the non-linear phase characteristics in modern domestic television receivers have given rise to an increased interest in the measurement of the group-delay characteristics of these receivers[9–11]. Consequently, the need for a simple measuring device has become urgent. In this article such a device is described. It is especially intended for use with intermediate-frequency amplifiers of television receivers; the equipment is expected to become commercially available this year.

Principle

The underlying principle of the instrument is well known; it was given by Brand and Nyquist in 1930. The high-frequency output voltage of a standard-signal generator (angular frequency ω) is amplitude modulated by a low-frequency sinusoidal voltage (angular frequency p) generated by an oscillator (see Fig. 1). The modulated signal passes the network under test and is subsequently detected. The phase shift between the detected signal and the modulating signal depends directly on the group-delay time of the network at the frequency ω. This follows from Fig. 2 in which the phase–frequency characteristic of the network under test has been depicted. For a small value of the low frequency p, the phase shift of each sideband component of the modulated signal equals $p\ d\phi/d\omega$. The phase of the amplitude modulation is shifted over this same angle; therefore, the phase difference indicated by the phase meter Ph.M. in Fig. 1 is

$$\Delta\phi = p\,\frac{d\phi}{d\omega} = p\ \tau_{gr},$$

in which equation the group-delay at the frequency ω is indicated by τ_{gr}.

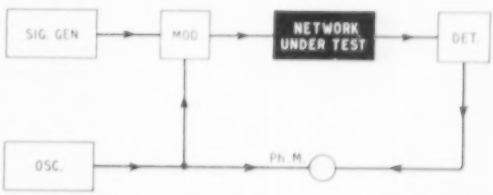

Fig. 1. *Principle of group-delay measurement. Sig. Gen. = standard signal generator; Mod. = modulator; Det. = detector; Osc. = oscillator; Ph.M. = phase meter.*

If the carrier frequency is varied, the change in the phase shift $\Delta\phi$ is proportional to the change in group delay. Consequently, the scale of the phase meter can be calibrated directly in group-delay time units, usually milli-micro-seconds (nano-seconds).

One of the main problems in designing such a measuring equipment is the necessary sensitivity

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for the phase meter. The group delay measured according to this principle is the average value of the actual group delay between $\omega - p$ and $\omega + p$. Obviously it is advantageous to choose the p-frequency as low as possible; the p-frequency should in any case be small compared to the frequency band in which the delay curve has to be measured. However, the ultimate phase shift $\Delta\phi$ is directly proportional to p; therefore a smaller value of the frequency p necessitates a high sensitivity of the phase meter. For instance with a frequency $f_p = 30$ kc/s, a group-delay variation of one milli-microsecond corresponds to a phase variation of 2×10^{-4} radian $= 0{\cdot}01$ degree.

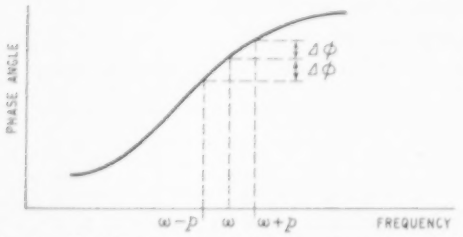

Fig. 2. *The phase of the sideband components is shifted over a phase angle $\Delta\phi = pd\phi/d\omega$ as compared with the phase of the carrier.*

Wobbulator Measurement

The group-delay measuring device to be described in this article traces the group-delay characteristic on the screen of an oscilloscope. This can be achieved by using a wobbulator generator as high-frequency voltage source and a phase meter, which gives a voltage or current output proportional to the phase shift to be measured. This method is in one aspect fundamentally simpler than the point-by-point method, but has also one fundamental drawback.

The advantage is that with a wobbulator measurement the sensitivity of the phase meter is no longer a problem, because the output signal of the phase meter is an a.c. signal. It can, therefore, be simply amplified to achieve the necessary sensitivity.

The disadvantage lies in the fact that with a wobbulator system the bandwidth of the output signal of the phase meter is substantially larger than with a point-by-point method. With a wobbulator having a repetition frequency of 50 c/s this bandwidth may be several thousand cycles, whereas with a point-by-point method the bandwidth can be reduced to one cycle or even less. As a consequence, the noise level will be much larger with a wobbulator system.

The signal at the detector output consists of a p-frequency voltage, the phase of which is modulated according to the magnitude of the group-delay at the instantaneous frequency. The magnitude of this phase modulation is directly proportional to the value of the p-frequency. Thus a high noise level can be counteracted by choosing a high p-frequency.

As a matter of fact, the p-frequency, for which a value of 30 kc/s was chosen in the preliminary experiments, had to be increased to 100 kc/s to achieve a satisfactory signal-to-noise ratio.

The phase modulation, which contains the information on the group-delay characteristic, can be derived from the phase-modulated p-frequency voltage by applying the latter to any conventional phase or frequency-detector circuit. Because, in this case, a reference p-frequency signal with constant phase is available, a phase detector of the type EQ80 was chosen. It was found that a preceding limiter stage was necessary to reduce the noise level to a minimum.

Necessity for a Constant Detector Level

From experiments with other delay meters it was known that the carrier-frequency level on the detector diode should be kept rigorously constant during the measurement of a group-delay characteristic. This necessity is due to the fact that, with most wideband amplifiers, the detector does not work fully linearly because the voltage on the detector is rather small. In a detector stage, the detector diode gives a certain amount of coupling between i.f. and video-frequency circuits. With a non-linear detector this coupling depends on the magnitude of the i.f. carrier voltage. As a consequence, the properties of the i.f. circuit depend to a certain extent on the signal level on the diode.

The effect on the measured group-delay characteristic is such that, with different signal levels on the diode, different phase shifts are introduced in the p-frequency signal. However, as a first approximation and at a given level, this phase shift does not vary throughout the frequency band considered. If, therefore, group-delay characteristics are measured at different, but constant, diode levels, the characteristics have the same shape but are only shifted in the vertical direction over different distances. As the absolute value of the group delay is of no importance for most purposes, this constant group-delay shift need not be considered.

It is obviously necessary to maintain a constant level on the diode during the measurement of a group-delay curve, because a varying level would cause a varying phase shift in the p-frequency signal during the measurement of a delay curve, which varying phase shift would be interpreted by the equipment as a change in group-delay time. Erroneous measurements would be the result.

The constant signal level on the diode can, with point-by-point measurements, easily be achieved by varying the output of the standard-signal generator. This can be done manually using the diode direct voltage as an indication.

With a wobbulator measurement, this amplitude control has obviously to be done by an automatic-gain-control device. For this purpose the p-frequency output signal of the detector is rectified to give a direct voltage which controls the gain of a radio-frequency amplifier stage inserted between the wobbulator and the modulator. The rectifier diode has a large bias voltage on its cathode so as to achieve a rigorous constant voltage on the i.f. detector.

An additional advantage of this automatic-gain-control system follows from the fact that the control voltage is a measure of the amplification of the network under test. By using a valve with suitable g_m–V_g curve in the controlled stage, the control voltage can be made roughly proportional to the logarithm of the gain of the network under test. By applying the control voltage to a second oscilloscope, the amplitude characteristic of the network under test will then be traced on a logarithmic scale. Thus both amplitude and delay characteristic are measured at the same time, which is a great convenience in the adjustment of filters.

wobbulator generator is fitted with a calibrated attenuator. A certain decrease of the wobbulator output, for instance 10 dB, causes the amplitude curve to shift as a whole over a corresponding distance in the vertical direction.

The delay meter can be used in combination with a conventional wobbulator generator. In most wobbulators a frequency-marker device enables the frequency axis to be calibrated. However, these markers are often so-called 'active' markers, consisting of a variable oscillator the frequency scale of which is accurately calibrated. Such an active marker is less suitable for a frequency marker with the delay meter for the following reason.

The signal of the wobbulator as well as the signal of the marker oscillator will both be amplitude modulated in the modulator stage with the p-frequency voltage (see Fig. 1); therefore, the input voltage of the network under test will consist of two carrier frequencies, each with two sidebands. One carrier with its sidebands is wobbulated through the frequency band considered; when it passes the constant frequencies of the marker signal with its sidebands, a beat frequency equal to the p-frequency will be formed at a number of discrete frequencies of the wobbulator signal.

At each of these frequencies, this extra p-

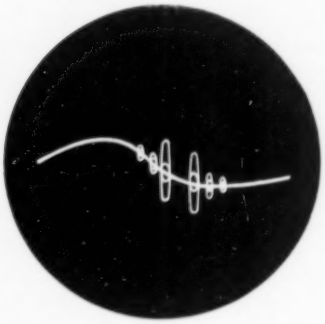

Fig. 3 (left). *An active frequency marker causes a complicated interference pattern in the group-delay curve on the oscilloscope.*

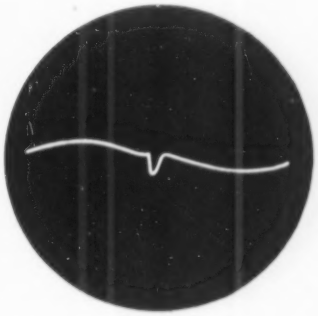

Fig. 4 (right). *The influence of an absorption frequency marker on the group-delay curve on the oscilloscope.*

Scale Calibration

The output of the phase meter (EQ80) is fed to the input terminals of the oscilloscope. In general, the sensitivity of an oscilloscope is not very accurately known; therefore a device enabling the calibration of the delay scale in terms of milli-microseconds per cm is necessary.

This calibration can be performed by introducing a known phase shift $\Delta\phi$ in the path of the p-frequency signal. This phase shift causes a vertical movement of the delay curve as a whole over a distance corresponding to a delay equal to $\Delta\phi/p$. This provides a direct calibration of the vertical scale on the oscilloscope.

The amplitude scale can be calibrated if the frequency component causes a sudden phase shift in the output of the detector, which gives a deviation in the delay characteristic on the oscilloscope. As a consequence, an active marker does not give one indication on the delay curve, but a discrete number spaced at mutual frequency differences equal to the p-frequency (see Fig. 3). Sharp frequency calibration is therefore impossible.

To enable a better frequency calibration, a passive absorption marker circuit has to be provided; such a marker consists of a tuned circuit of high quality which is loosely coupled to the radio-frequency circuit. It causes a dip in the delay curve of the shape depicted in Fig.

4 and, at the same time, a dip in the amplitude curve; however this dip is much smaller than the dip in the delay characteristic. Increased coupling with the absorption circuit obviously increases both dips to the same extent, so a compromise has to be made, between a somewhat too small dip in the amplitude characteristic and a too large dip in the delay curve.

Description of the Complete Instrument

The circuit diagram is given in Fig. 5. The oscillator triode, which generates the *p*-frequency (100 kc/s in this instrument) is the triode of a valve ECH81 (V_8). Part of the generated voltage is directly fed to one of the control grids of the EQ80 (V_{10}). Another part of the oscillator voltage is amplified in the heptode part of

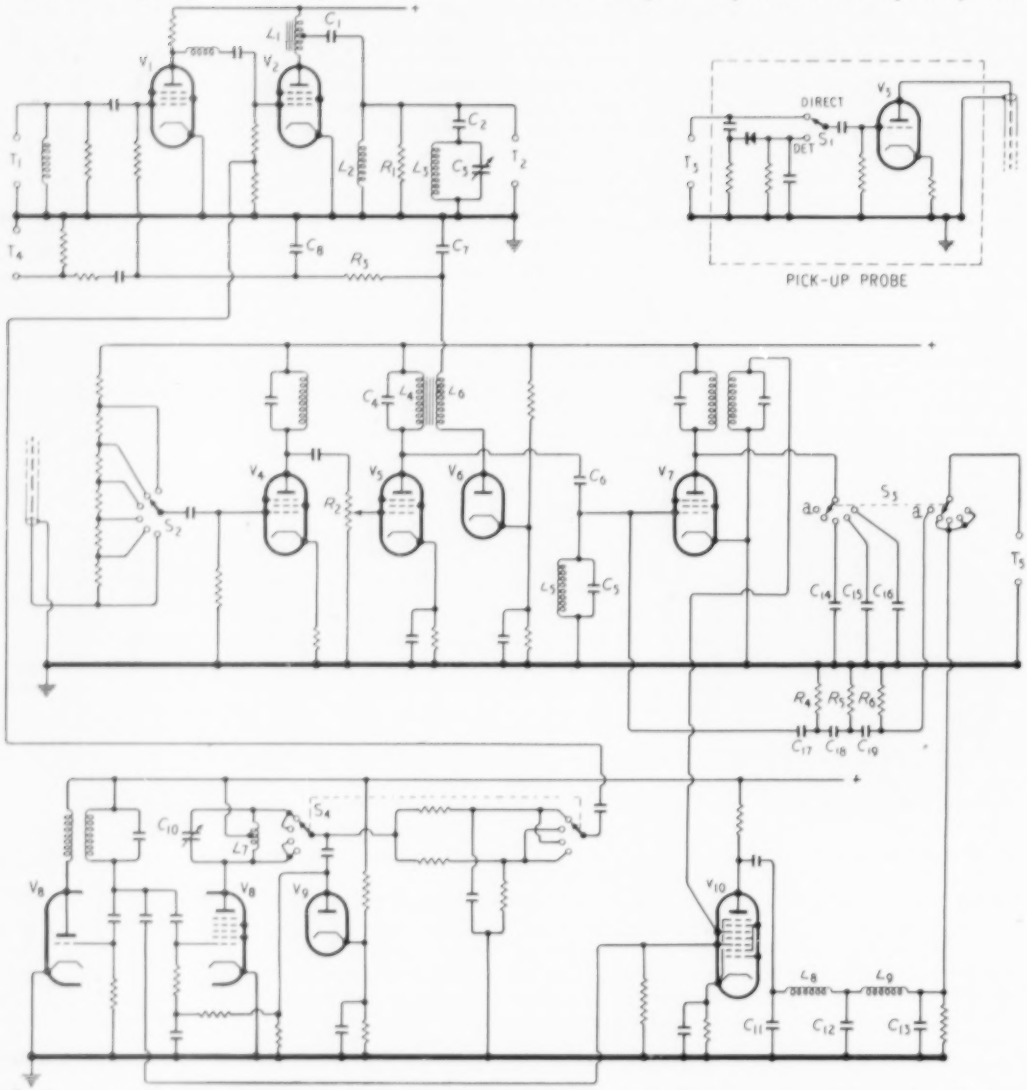

Fig. 5. Simplified circuit diagram of the group-delay meter. Upper left, radio-frequency amplifier and modulator; upper right, pick-up probe; centre, p-frequency amplifier; lower left, p-frequency oscillator and phase-shifter; lower right, phasemeter EQ80; terminals, T_1—radio-frequency input from wobbulator; T_2—radio-frequency output to network under test; T_3—radio-frequency or p-frequency input from network under test; T_4—low-frequency output to amplitude-characteristic oscilloscope; T_5—low-frequency output to delay-characteristic oscilloscope. Valves: V_1, V_2, EF95; V_3, EC70; V_4, V_5, V_7, EF83; V_6, V_9, EA50; V_8, ECH81; V_{10}, EQ80.

valve V_8 and passes subsequently a phase-shifting device which allows the introduction of 4×90 degrees phase shift in discrete steps (S_7), as well as continuous phase shift over about 100 degress (C_{10}).

The necessity for this phase-shifting device is given by the fact that the EQ80 has only a limited range of phase angles in which the anode current is linearly proportional to the phase difference between the two input voltages.

As the network under test may introduce any amount of constant phase shift in the 100-kc/s signal, it is essential to have a phase shifter in the delay meter which can counteract a possible large phase shift.

In this apparatus, the continuous phase shift is brought about by detuning the anode circuit L_7C_{10} of the heptode V_8. The 4×90 degrees phase shift is achieved with simple resistance-capacitance sections in combination with a $+$ 180 degree switch connected to the anode circuit L_7C_{10}. Obviously, the output voltage of such a simple phase-shifting device is not constant. An automatic-gain-control circuit, comprising the diode V_9, stabilizes the output voltage of the phase shifter. This output voltage is fed to the modulator valve V_2.

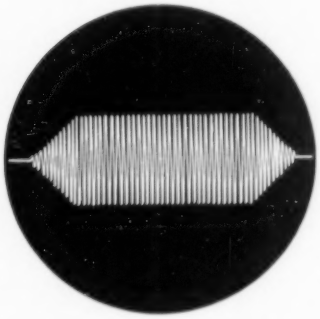

Fig. 6. *Image on the oscilloscope in monitoring position of switch S_3.*

The radio-frequency signal from the wobbulator generator enters the apparatus at the terminal T_1 (input impedance 75 Ω) and is subsequently amplified in the gain-controlled valve V_1 and amplitude-modulated by the p-frequency signal in valve V_2. The anode impedance of this modulator stage consists of the inductance L_1, which is in parallel resonance with the anode capacitance at the centre frequency of the pass-band. The output terminal T_2 is connected to a tap on L_1; the resistance R_1 serves as terminating resistance for the output cable as well as damping resistance for the anode circuit of the modulator stage. The control-grid impedance of valve V_1, the coupling network between the valves V_1 and V_2 and the transformer L_1 have been designed so as to give a flat amplitude characteristic as well as a flat group-delay curve in the frequency range from 20–45 Mc/s. The capacitance C_1 and the inductance L_2 act as a high-pass filter, thus suppressing the 100-kc/s signal. The absorption frequency marker L_3C_3 is coupled to the output lead by way of a small capacitance C_2.

In many cases the network, of which the input is connected to the output-terminal T_2, will contain a detector stage at the output of which the 100-kc/s signal will be directly available.

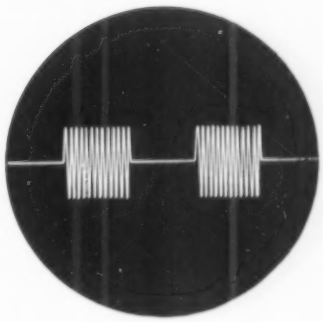

Fig. 7. *Output voltage of the wobbulator generator G.M.2889.*

However, it also often occurs that no such detector is present, for instance if measurements are made on a single stage or a part of an i.f. amplifier. In these cases a separate detector is necessary. Therefore, the pick-up probe (see upper right-hand corner of Fig. 5) contains a switch S_1 with two positions. In position 'direct' a 100-kc/s input signal is directly fed to the grid of the valve V_3. In the position 'det.' an i.f. input signal is first demodulated by a crystal diode, the detected 100-kc/s signal being subsequently fed to the same valve V_3.

After passing V_3 in the probe, the signal is amplified by V_4 and V_5 in the delay-meter itself. The switch S_2 permits a discontinuous control of the total gain in steps of roughly 10 dB, while potentiometer R_2 provides a continuous adjustment of the gain.

The anode impedance of valve V_5 consists of a capacitively-coupled bandfilter L_4C_4, C_6, L_5C_5. The voltage for the p-frequency amplitude detector (the diode V_6) is taken from an inductance L_6, which is tightly magnetically-coupled to the inductance L_4 of the bandfilter. The detector diode V_6 has a large d.c. bias voltage on its cathode so as to achieve a very rigorous gain control. The rectified voltage is fed through the inductance L_6 and the low-pass filter $C_7R_3C_8$ to the control grid of the gain-controlled stage V_1. A part of this voltage can be taken from the terminal T_4 to feed the amplitude-characteristic oscilloscope.

The secondary circuit of the bandfilter L_4C_4, C_6, L_5L_5 is connected to the control grid of limiter valve V_7. The anode of this valve is coupled through a bandfilter to the second control-grid of the phase-detector valve V_{10}.

The low-frequency output signal of this phase detector is taken through a low-pass filter $C_{11}L_8C_{12}L_9C_{13}$ to the output terminal T_5. The voltage at this terminal is such that it can be fed directly to an oscilloscope with conventional sensitivity (20 mV_{eff}/cm).

Calibration of the delay axis can be done with switch S_3, which switch connects different capacitances in parallel to the input terminals of the bandfilter between valves V_7 and V_{10}. The detuning of this bandfilter causes a certain phase shift, which corresponds in the three positions of switch S_3 to delay variations of 25, 100 and 250 milli-microseconds, respectively.

In the extra position (a) of switch S_3, the input of the oscilloscope is connected by way of a high-pass filter $C_{17}R_4C_{18}R_5C_{19}R_6$ to the grid of the limiter valve V_7 so as to make the 100 kc/s-signal visible on the oscilloscope. The high-pass filter is necessary to reject low-frequency voltages due to level variations of the 100-kc/s signal. The level of the 100-kc/s signal is constant as long as the a.g.c. system works effectively. Outside the pass-band of the network under test, however, the total amplification can decrease to such an extent that the a.g.c. system can no longer counteract this decrease. At these carrier frequencies, the level on the i.f. detector may therefore vary considerably, which might cause substantial errors in the group-delay curve as indicated on the oscilloscope.

For an accurate measurement the i.f. carrier level at the detector should therefore be constant, in which case the 100-kc/s level on the grid of the limiter valve will also be constant. This can be monitored in the extra position of switch S_3. The image on the oscilloscope should resemble the picture of Fig. 6. The group-delay curve can be trusted over the frequency range where the 100-kc/s signal has a constant amplitude.

Performance of the Equipment

The delay meter can be used with any wobbulator generator; we combined it with the Philips wobbulator GM 2889 and used it extensively in the design of i.f. amplifiers for television receivers[11]. The output voltage of the wobbulator GM 2889, the sweep-repetition frequency of which is 50 c/s, is in one half of the sweep period a wobbulated signal, whereas in the other half of the period the output is nil (see Fig. 7). As a consequence a horizontal zero axis is drawn on both oscilloscopes, which is of great help in the estimation of the non-flatness of the measured characteristics as well as the calibration process.

A measuring set-up is shown in Fig. 8; the amplitude and group-delay characteristics of the

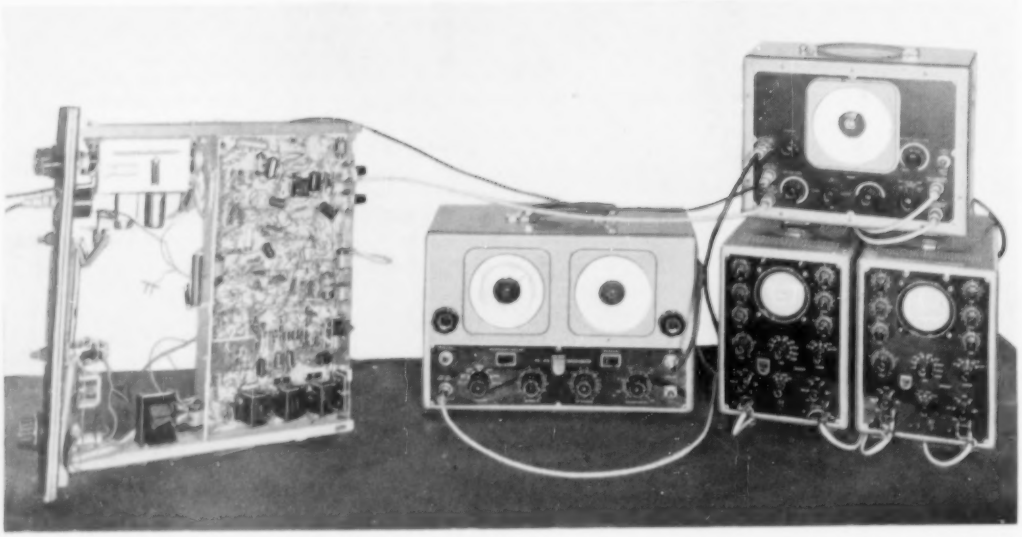

Fig. 8. *Set-up for measuring the steady-state characteristics of an i.f. amplifier of a television receiver (left). Wobbulator generator type GM 2889 in the centre. Delay-measuring equipment type GM 2894 on top of the oscilloscopes. Left-hand oscilloscope gives amplitude curve, right-hand oscilloscope gives group delay curve. Sensitivities for the curves on the oscilloscopes: 6 dB/cm for amplitude, 50 × 10^{-9} sec/cm for delay.*

receivers under test appear on the screens of the left- and right-hand oscilloscopes, respectively.

In general, the technique of measuring a group-delay curve is somewhat more difficult than the classical measurement of an amplitude curve in that it is easier disturbed by interfering signals, more vulnerable to the overloading of valves, etc. This can well be understood, because a group-delay measurement is a very sensitive measurement. Some possible error sources have been described elsewhere[1,2]; measures have been taken in the design of this equipment to eliminate these error sources as much as possible (for instance the a.g.c. system). The user of the instrument on the other hand should be well aware of the possible consequences of the disturbing external influences mentioned above.

The apparatus is primary designed for measurements on i.f. amplifiers of television receivers; therefore the frequency band has been fixed at 20–45 Mc/s. The sensitivity depends ultimately on the noise level, because the amplification in the oscilloscope can be arbitrarily increased. The inherent noise level of the delay meter is such that group-delay variations of below one milli-microsecond can be distinguished. A much smaller sensitivity is sufficient for measurements with television receivers; our practice is to adjust the vertical amplification of the oscilloscopes to a sensitivity of 50 milli-microseconds per cm for the delay oscilloscope and of 6 dB/cm for the amplitude oscilloscope. With measurements on high-gain amplifiers, the noise introduced in such an amplifier may disturb the measurements even at this reduced sensitivity. It is then necessary to decrease the gain of the amplifier to make the measurement possible at all.

It may be mentioned that an additional unit has been developed, which enables the tracing of video-frequency delay and amplitude characteristics on oscilloscopes (frequency range 300 kc/s–10·0 Mc/s). We hope to describe this equipment in a later publication.

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PHASE-ANGLE MEASUREMENT

C.R. Tube Methods

By C. H. Vincent, M.Sc., A.Inst.P., A.M.I.E.E.

SUMMARY.—The paper discusses the method of phase-angle measurement due to Fleming, in which X and Y deflections are adjusted for equality, and the phase difference is deduced from the proportions of the resulting ellipse. A thorough analysis is made of the geometrical factors which might lead to error in this method, and it is found that, provided the method is modified slightly to take certain reasonable precautions, it is an accurate and practical one at all phases, in spite of certain criticisms which have been made.

Introduction

IT is well known that the phase difference between two sinusoidal voltages of the same frequency may be determined in several ways[1,2,3] from measurements of the ellipse that appears when the two voltages are applied to the deflection plates of a cathode-ray tube. One such method[1,4] is to adjust the relative amplitudes of the voltages so that the peak X and Y deflections are equal, and to measure the major and minor diameters of the ellipse. The phase difference is then 2α, where

$$\alpha = \pm \tan^{-1} \frac{D_2}{D_1} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where D_1 is the major or minor diameter that lies within the 1st and 3rd quadrants, and D_2 that which lies within the 2nd and 4th (see Fig. 1). This method has the practical advantages that it

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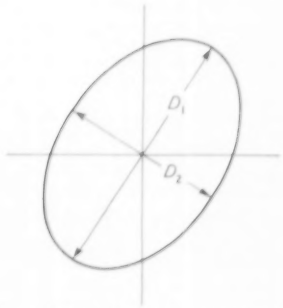

Fig. 1. *Measurement of D_1 and D_2. The scale is orientated for maximum and minimum measurement on a line through the origin.*

is simple and direct, and that the measurements are made with the trace crossing the scale at right angles, permitting greater accuracy. Since

$$\frac{d}{d\alpha} \tan \alpha = \sec^2 \alpha > 1$$

an accurate determination of $\tan \alpha$ gives an

accurate value of α at all phases. [See also Equ. (10)]. The sign of the phase difference is not indicated by the ellipse, and must be determined from some other consideration, whatever measurements are made.

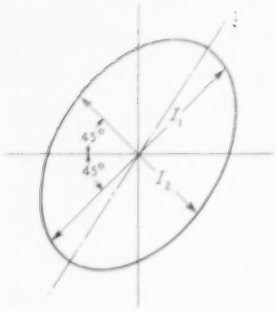

Fig. 2. Measurement of I_1 and I_2. The scale is orientated at 45° to the axes on a line through the origin and the intercepts are measured.

Discussion

This method of measurement has been criticized by Benson[5] on the grounds that appreciable error might be caused by the residual inequality of the deflection amplitudes or by incorrect positioning of the scale used. However, if a detailed analysis is made, taking account of any such residual inequality, it is found that the

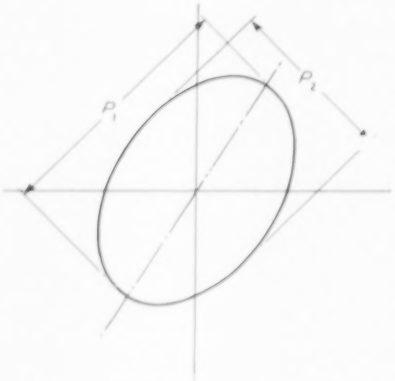

Fig. 3. Measurement of P_1 and P_2. The scale is orientated at 45° and the distances between the tangents to the ellipse at 45° to the axes are measured.

error due to this inequality is, in general, a second-order effect [see Equ. (4)], and can therefore be kept very small with reasonable care in the amplitude adjustment. Moreover, when this adjustment has been made, the axes of the ellipse coincide closely in general with the radials from the origin at 45° to the deflection axes. Further analysis shows that either the intercepts I_1, I_2 of the ellipse on these 45° radials (see Fig. 2) or the perpendicular distances P_1, P_2 between the tangents to the ellipse which are parallel to these radials (see Fig. 3) may be substituted for D_1 and D_2 in the ratio giving $\tan \alpha$. The error due to any difference between the deflection amplitudes is again normally of the second order [see Equs. (5a), (5b), (6a), (6b)], and may therefore be kept very small. Hence scales fixed at 45° to the deflection axes may be used, if necessary, to avoid error due to incorrect scale orientation, the scales being used appropriately to measure I_1, I_2 or P_1, P_2 as desired.

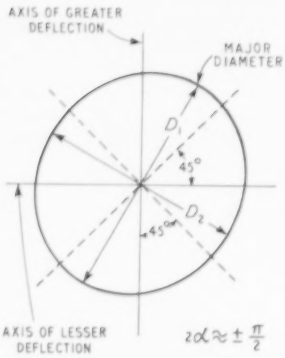

Fig. 4. When D_1 and D_2 become inaccurate, I_1, I_2, P_1 and P_2 are accurate. The ellipse approaches circular form and the major and minor axes may shift towards the deflection axes.

Although either D, I, or P may be used, in general, with only second-order errors, the equations referred to above show that there is one phase angle where each of these measurements becomes relatively inaccurate, and it is therefore desirable to use one of the other two in this vicinity, although the error will still be small, unless the equality adjustment is very poor. These conditions are illustrated with an exaggerated amplitude difference in Figs. 4, 5 and 6. It is clear from these figures that where the errors occur they can be anticipated intuitively, apart from the analysis. The procedure for this method is summarized below, and should give an accurate and practical measurement.

Furthermore, provided that the conditions

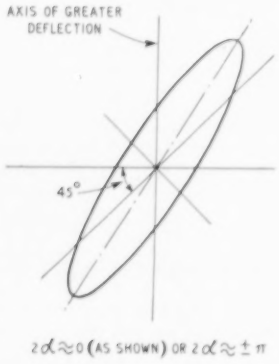

Fig. 5. When I_{major} becomes inaccurate, D_1, D_2, I_{minor} and P_{major} are accurate. The ellipse becomes long and narrow and the major axis may shift off the 45° radial.

described as leading to inaccuracy are avoided, errors in the measurement of D, I, or P, due to slight displacement or misorientation of the measuring scale, are in all cases second-order, and can therefore be kept very small, with reasonable care. This is because the measurements D and I are made at or near values which are stationary with respect to small changes in the position or orientation of the scale. Measurements P are also made at or near stationary values with respect to changes in orientation, and are independent of position.

A first-order effect on the ratio determining $\tan \alpha$ will be caused by any error in the relative orientation of the deflection axes of the cathode-ray tube itself [see Equs. (7a), (7b)], but this error is normally small. It could, if necessary, be compensated for by reversing the phase of one of the voltages thereby, in effect, interchanging the 1st and 3rd quadrants with the 2nd and 4th, and taking the geometrical mean of the ratios thus obtained [see Equs. (8a), (8b), (9)]. Further sources of possible error, against which the usual precautions should be taken, are parallax, non-linear deflection to voltage relationships, unwanted phase shifts of the voltages applied to the deflector plates, and the presence of harmonics in these voltages.

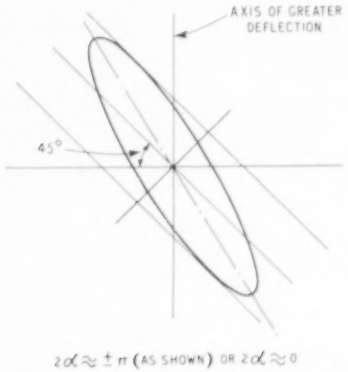

Fig. 6. *When P_{minor} becomes inaccurate, D_1, D_2, I_{minor} and P_{major} are accurate. The ellipse becomes long and narrow and the corresponding 45° tangents may become separated by more than the minor diameter.*

Summary of Procedure

(1) Adjust the deflection amplitudes to equality.

(2) If the ellipse is approximately circular, calculate the phase difference as

$$2\alpha = \pm 2 \tan^{-1} \frac{Q_2}{Q_1} \quad \ldots \quad \ldots \quad (2)$$

where $Q_1 = I_1$ or P_1 and $Q_2 = I_2$ or P_2 respectively.

(3) If the ellipse is markedly elongated, calculate the phase difference as above but with $Q_1 = D_1$, or I_1 (if minor), or P_1 (if major), and with $Q_2 = D_2$, or I_2 (if minor), or P_2 (if major), according to the positions of the major and minor axes.

(4) In other cases, use either D, I, or P.

(5) If desired, compensate for faulty relative orientation of the c.r. tube deflection axes by reversing the phase of one voltage (interchanging suffixes 1 and 2 above), and taking the geometrical mean of the ratios as $\tan \alpha$.

Acknowledgment

I am most grateful to my colleagues Dr. W. Fishwick, Dr. H. M. Melvin, and Mr. T. S. M. Maclean for helpful comment and criticism.

APPENDIX 1

Summary of the Mathematical Results

Consider deflections given by

$$x = (A + \delta) \cos (\omega t + \alpha) \quad \ldots \quad \ldots \quad \ldots \quad (3a)$$

$$y = (A - \delta) \cos (\omega t - \alpha) \quad \ldots \quad \ldots \quad \ldots \quad (3b)$$

where 2δ is the residual difference in amplitude, and the other symbols have their usual significance. Then it is found that

$$D^2_{maj.}\ D^2_{min} = 4(A^2 + \delta^2) \pm 2[2A^4 + 12A^2\delta^2 + 2\delta^4 + 2(A^2 - \delta^2)^2 \cos 4\alpha]^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

giving

$$D_1^2 \approx 8A^2 \cos^2 \alpha \quad \text{for small } \delta$$

and

$$D_2^2 \approx 8A^2 \sin^2 \alpha \quad \text{for small } \delta$$

D_1 being the major axis for $-\pi/2 < 2\alpha < \pi/2$ and D_2 for $-\pi < 2\alpha < -\pi/2$ or $\pi/2 < 2\alpha < \pi$. The error is of order δ^2 in both cases, except when $\cos 4\alpha = -1$ so that the term in A^4 disappears from within the square-root sign. This occurs when $2\alpha = \pm \pi/2$, and the ellipse approximates to a circle.

Also

$$I_1^2 = \frac{4(A^2 - \delta^2)^2 \sin^2 2\alpha}{A^2 + \delta^2 - (A^2 - \delta^2) \cos 2\alpha} \quad \ldots \quad \ldots \quad (5a)$$

$$\approx 8A^2 \cos^2 \alpha \quad \text{for small } \delta$$

and

$$I_2^2 = \frac{4(A^2 - \delta^2)^2 \sin^2 2\alpha}{A^2 + \delta^2 + (A^2 - \delta^2) \cos 2\alpha} \quad \ldots \quad \ldots \quad (5b)$$

$$\approx 8A^2 \sin^2 \alpha \quad \text{for small } \delta$$

the error being of order δ^2 in each case, except when the term in A^2 in the denominator vanishes. This occurs when the intercept concerned is the major one and when $\cos 2\alpha = 1$ (i.e., $2\alpha = 0$) for I_1, and when $\cos 2\alpha = -1$ (i.e. $2\alpha = \pm \pi$) for I_2. At all of these phases the ellipse degenerates into a segment of a straight line.

Also

$$P_1^2 = 4(A^2 + \delta^2) + 4(A^2 - \delta^2) \cos 2\alpha \quad \ldots \quad (6a)$$

$$\approx 8A^2 \cos^2 \alpha \quad \text{for small } \delta$$

and

$$P_2^2 = 4(A^2 + \delta^2) - 4(A^2 - \delta^2) \cos 2\alpha \quad \ldots \quad (6b)$$

$$\approx 8A^2 \sin^2 \alpha \quad \text{for small } \delta$$

the error being of order δ^2 and negligible in each case except when the term in A^2 vanishes. This occurs when the perpendicular distance P concerned is the minor one and when $\cos 2\alpha = -1$ (i.e., $2\alpha = \pm \pi$) for P_1, and when $\cos 2\alpha = 1$ (i.e., $2\alpha = 0$) for P_2. These are again the phases at which the ellipse degenerates into a segment of a line.

If the angle in the 1st quadrant between the X and Y axes of the cathode-ray tube is $(90 - 2\theta)$ degrees, then

$$Q_1' \approx Q_1 (\cos \theta - \sin \theta) \quad \ldots \quad \ldots \quad \ldots \quad (7a)$$

and

$$Q_2' \approx Q_2 (\cos \theta - \sin \theta) \quad \ldots \quad \ldots \quad \ldots \quad (7b)$$

where Q_1' and Q_2' refer to the actual values of D, I, or P as measured, and Q_1 and Q_2 refer to the corresponding

values which they would have if the axes were perpendicular.

Also

$$Q_1'' \approx Q_1 (\cos\theta - \sin\theta) \quad \ldots \quad \ldots \quad \ldots \quad (8a)$$

and

$$Q_2'' \approx Q_2 (\cos\theta + \sin\theta) \quad \ldots \quad \ldots \quad \ldots \quad (8b)$$

where Q_1'' and Q_2'' refer to the corresponding measurements with the phase of one voltage reversed and with the suffixes transferred accordingly.

Hence

$$\sqrt{\frac{Q_2'}{Q_1'} \cdot \frac{Q_2''}{Q_1''}} \approx \frac{Q_2}{Q_1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

It is of interest to note that if the measurements of Q_1 and Q_2 are subject to independent random errors δQ_1 and δQ_2 such that $(\delta Q_1)^2 = (\delta Q_2)^2 = \sigma^2$, as is reasonable to assume, σ being a constant, and if these errors cause an error $\delta\alpha$ in α, then

$$(\delta\alpha)^2 = \frac{\sigma^2}{8A^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

which is independent of the phase being measured. In practice, therefore, the error in phase from this cause is likely to be of the same order at all phases.

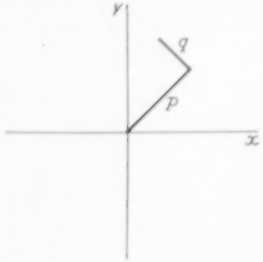

Fig. 7. *Definition of co-ordinates p and q.*

APPENDIX 2

Proofs

If $A + \delta = B$ and $A - \delta = C$, then from equations (3a), (3b)

$$\begin{aligned} x^2 + y^2 &= B^2\cos^2(\omega t + \alpha) + C^2\cos^2(\omega t - \alpha) \\ &= \frac{B^2}{2}\left[1 + \cos 2(\omega t + \alpha)\right] + \\ &\quad \frac{C^2}{2}\left[1 + \cos 2(\omega t - \alpha)\right] \\ &= \frac{B^2 + C^2}{2} + \frac{B^2 + C^2}{2}\cos 2\omega t \cos 2\alpha - \\ &\quad \frac{B^2 - C^2}{2}\sin 2\omega t \sin 2\alpha \\ &= \frac{B^2 + C^2}{2} + \left[\left(\frac{B^2 + C^2}{2}\right)^2\cos^2 2\alpha + \right. \\ &\quad \left.\left(\frac{B^2 - C^2}{2}\right)^2\sin^2 2\alpha\right]^{\frac{1}{2}} \\ &\quad \times \cos\left[2\omega t + \tan^{-1}\frac{B^2 - C^2}{B^2 + C^2}\tan 2\alpha\right] \end{aligned}$$

Hence

$$\begin{aligned} D^2_{maj}, D^2_{min} &= 4(x^2 + y^2)_{max}, 4(x^2 + y^2)_{min} \\ &= 2(B^2 + C^2) \pm \\ &\quad 4\left[\left(\frac{B^2 + C^2}{2}\right)^2\cos^2 2\alpha + \left(\frac{B^2 - C^2}{2}\right)^2\sin^2 2\alpha\right]^{\frac{1}{2}} \\ &= 2(B^2 + C^2) \pm 2[B^4 + 2B^2C^2\cos 4\alpha + C^4]^{\frac{1}{2}} \\ &= 4(A^2 + \delta^2) \pm \\ &\quad 2[2A^4 + 12A^2\delta^2 + 2\delta^4 + 2(A^2 - \delta^2)^2\cos 4\alpha]^{\frac{1}{2}} \end{aligned}$$

as stated above [Equ. (4)].

From the original deflection equations (3a) and (3b),

$$\begin{aligned} \frac{x}{B} + \frac{y}{C} &= \cos(\omega t + \alpha) + \cos(\omega t - \alpha) \\ &= 2\cos\omega t \cos\alpha \end{aligned}$$

and similarly

$$\frac{x}{B} - \frac{y}{C} = -2\sin\omega t \sin\alpha$$

Eliminating ωt, we get

$$\frac{\left(\frac{x}{B} + \frac{y}{C}\right)^2}{4\cos^2\alpha} + \frac{\left(\frac{x}{B} - \frac{y}{C}\right)^2}{4\sin^2\alpha} = 1$$

giving

$$\frac{x^2}{B^2} + \frac{y^2}{C^2} - \frac{2xy}{BC}\cos 2\alpha = \sin^2 2\alpha \quad \ldots \quad \ldots \quad (11)$$

For the intercept on the 45° radial in the 1st quadrant, $x_1 = y_1 = \frac{I_1}{2\sqrt{2}}$. Hence

$$\begin{aligned} x_1^2\left(\frac{1}{B^2} + \frac{1}{C^2} - \frac{2}{BC}\cos 2\alpha\right) &= \sin^2 2\alpha \\ \therefore I_1^2 = 8x_1^2 &= \frac{8B^2C^2\sin^2 2\alpha}{B^2 + C^2 - 2BC\cos 2\alpha} \\ &= \frac{4(A^2 - \delta^2)^2\sin^2 2\alpha}{A^2 + \delta^2 - (A^2 - \delta^2)\cos 2\alpha} \end{aligned}$$

as stated above [Equ. (5a)]. I_2 may be derived similarly [Equ. (5b)].

If equation (11) above is transformed to co-ordinates p, q, at 45° to the original ones (see Fig. 7) we have

$$x = \frac{p - q}{\sqrt{2}}$$

and

$$y = \frac{p + q}{\sqrt{2}}$$

giving

$$\begin{aligned} &\frac{p^2 - 2pq + q^2}{2B^2} + \frac{p^2 + 2pq + q^2}{2C^2} - \frac{(p^2 - q^2)\cos 2\alpha}{BC} \\ &\quad = \sin^2 2\alpha \end{aligned}$$

i.e.,

$$\begin{aligned} &p^2\left(\frac{1}{B^2} + \frac{1}{C^2} - \frac{2\cos 2\alpha}{BC}\right) + q^2\left(\frac{1}{B^2} + \frac{1}{C^2} + \frac{2\cos 2\alpha}{BC}\right) \\ &\quad + 2pq\left(\frac{1}{C^2} - \frac{1}{B^2}\right) = 2\sin^2 2\alpha \end{aligned}$$

Differentiating with respect to q and putting $\frac{dp}{dq} = 0$ for maximum or minimum p, we get

$$\frac{q}{p} = -\frac{\left(\frac{1}{C^2} - \frac{1}{B^2}\right)}{\left(\frac{1}{B^2} + \frac{1}{C^2} + \frac{2\cos 2\alpha}{BC}\right)}$$

and substituting for q,

$$\begin{aligned} p^2\left[\frac{1}{B^2} + \frac{1}{C^2} - \frac{2\cos 2\alpha}{BC} + \frac{\left(\frac{1}{C^2} - \frac{1}{B^2}\right)^2}{\left(\frac{1}{B^2} + \frac{1}{C^2} + \frac{2\cos 2\alpha}{BC}\right)}\right. \\ \left. - \frac{2\left(\frac{1}{C^2} - \frac{1}{B^2}\right)^2}{\left(\frac{1}{B^2} + \frac{1}{C^2} + \frac{2\cos 2\alpha}{BC}\right)}\right] = 2\sin^2 2\alpha \end{aligned}$$

i.e.,

$$\begin{aligned} p^2\left[\frac{1}{B^2} + \frac{1}{C^2} - \frac{2\cos 2\alpha}{BC} - \frac{\left(\frac{1}{C^2} - \frac{1}{B^2}\right)^2}{\left(\frac{1}{B^2} + \frac{1}{C^2} + \frac{2\cos 2\alpha}{BC}\right)}\right] \\ = 2\sin^2 2\alpha \end{aligned}$$

i.e.,

$$p^2 \frac{\frac{4}{B^2C^2}(1 - \cos^2 2\alpha)}{\frac{1}{B^2} + \frac{1}{C^2} + \frac{2\cos 2\alpha}{BC}} = 2\sin^2 2\alpha$$

$$\therefore p^2 = \frac{B^2 + C^2 + 2BC\cos 2\alpha}{2}$$

and since this is for maximum or minimum p

$$P_1^2 = 4p^2$$
$$= 4(A^2 + \delta^2) + 4(A^2 - \delta^2)\cos 2\alpha$$

as stated above [Equ. (6a)]. P_2 may be derived similarly [Equ. (6b)].

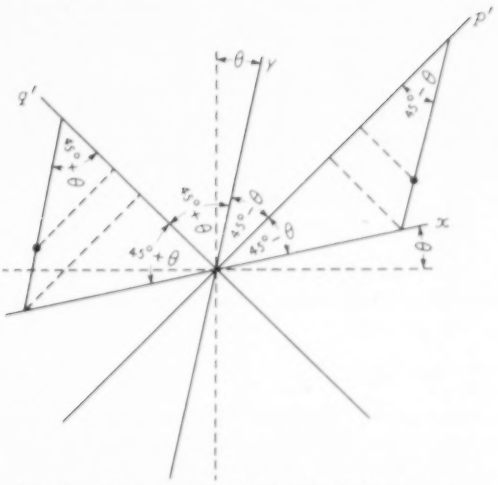

Fig. 8. *Effect of non-perpendicular deflection axes.*

If the X and Y deflection axes of the c.r. tube are not necessarily exactly at right angles, consider the transformation to co-ordinates p' and q' with axes bisecting the deflection axes (see Fig. 8). The p' and q' axes are then at right angles to each other. If the angle between the X and Y axes in the 1st quadrant is $(90-2\theta)$ degrees,

$$p' = x\cos(45 - \theta)^\circ + y\cos(45 - \theta)^\circ$$

and $$q' = -x\cos(45 + \theta)^\circ + y\cos(45 + \theta)^\circ$$

Hence, if p and q respectively denote the values which p' and q' would have if the axes were at right angles, for any given x and y, then

$$p' = p\frac{\cos(45 - \theta)^\circ}{\cos 45^\circ}$$

and $$q' = q\frac{\cos(45 + \theta)^\circ}{\cos 45^\circ}$$

For any measurement Q_1' which is made near the p axis, the corresponding co-ordinate q_1' is small compared with the corresponding co-ordinate p_1', and hence

$$Q_1'^2 = 4(p_1'^2 + q_1'^2)$$
$$= 4p_1^2\frac{\cos^2(45 - \theta)^\circ}{\cos^2 45^\circ} + 4q_1^2\frac{\cos^2(45 + \theta)^\circ}{\cos^2 45^\circ}$$
$$\approx 4(p_1^2 + q_1^2)\frac{\cos^2(45 - \theta)^\circ}{\cos^2 45^\circ}$$
$$\approx Q_1^2\frac{\cos^2(45 - \theta)^\circ}{\cos^2 45^\circ}$$

i.e., $$Q_1' \approx Q_1\frac{\cos(45 - \theta)^\circ}{\cos 45^\circ}$$
$$\approx Q_1\frac{\cos 45^\circ\cos\theta^\circ + \sin 45^\circ\sin\theta^\circ}{\cos 45^\circ}$$
$$\approx Q_1(\cos\theta^\circ + \sin\theta^\circ)$$

as stated above [Equ. (7a)]. Equations (7b), (8a), and (8b) can be derived similarly.

Regarding equation (10), if $\alpha = \tan^{-1}\frac{Q_2}{Q_1}$, then we have for any small δQ_1 and δQ_2

$$\tan^{-1}\frac{Q_2 + \delta Q_2}{Q_1 + \delta Q_1} \approx \alpha + \frac{1}{\sec^2\alpha}\left(\frac{\delta Q_2}{Q_1} - \frac{Q_2\cdot\delta Q_1}{Q_1^2}\right)$$

$$\therefore \delta\alpha \approx \frac{1}{\sqrt{8}\sec^2\alpha}\left(\frac{\delta Q_2}{A\cos\alpha} - \frac{A\cdot\delta Q_1\sin\alpha}{A^2\cos^2\alpha}\right)$$
$$\approx \frac{\delta Q_2}{A}\cdot\frac{\cos\alpha}{\sqrt{8}} - \frac{\delta Q_1}{A}\frac{\sin\alpha}{\sqrt{8}}$$

$$\therefore (\delta\alpha)^2 = \left(\frac{\delta Q_2}{A}\right)^2\frac{\cos^2\alpha}{8} + \left(\frac{\delta Q_1}{A}\right)^2\frac{\sin^2\alpha}{8}$$
$$= \frac{1}{8}\left(\frac{\sigma}{A}\right)^2$$

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VISUAL DETECTABILITY OF SIGNALS IN NOISE

The Effect of "Contrast"

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SUMMARY.—The effect of 'contrast' (in the form of a bias) on the detectability of signals displayed for visual observation in a noise background is very controversial, as many different considerations arise in different applications. This paper is concerned with the effect of such contrast on a particular derived signal-to-noise-ratio criterion (R_{B2}), and it is shown that the results obtained agree, at least qualitatively, with experimental results on probability of detection in very simple systems.

1. Introduction

SUBJECTIVE experiments[1] in the laboratory have shown reasonable amounts of contrast* to have little, or no, effect on the probability of detection. A theoretical treatment[4], which assumed simplified probability distributions, has shown that this result was not unexpected and that contrast would only have a significant effect when the number of Just Noticeable Differences of intensity available on the display was small, or the contrast very large. The calculation of the effect of contrast on detectability criteria†, using actual probability distributions has not hitherto been achieved, since long numerical calculations were involved. However, a recent internal report by Mr. F. A. J. Ford drew the author's attention to the fact that one of the integrals requiring solution occurred in other fields and had been important enough to employ a computing staff for numerical evaluation. With this inspiration an associated integral has been computed over a very small range and has enabled a few results to be obtained—sufficient to be indicative of the trend.

2. Theory

The criterion of output signal/noise ratio used most widely in theoretical assessments of various circuits is R_{B2} (see reference 2), which is defined as $\dfrac{\text{change in mean level on application of signal}}{\text{r.m.s. fluctuation with signal absent}}$.

The evaluation of this for any particular case, involves the calculation of the mean level and r.m.s. fluctuation in the absence of the signal and the mean level when the signal is present. The former presents little difficulty and it is only in the latter problem that the integration involves laborious computation.

Rice[3] shows that the probability density of the envelope of the combination of a pure sine wave and Gaussian random noise is given by

$$p(R) = \frac{R}{\psi_0} \exp.\left[-\frac{R^2+Q^2}{2\psi_0}\right] I_0\left(\frac{RQ}{\psi_0}\right)$$

where R = amplitude
ψ_0 = mean square of the noise
Q = peak amplitude of signal
I_0 = is a modified Bessel function

This equation can be normalized for simplicity by letting $x = \dfrac{R}{\sqrt{2\psi_0}}$ and $\dfrac{Q}{\sqrt{2\psi_0}} = y =$ signal/noise ratio. We wish to express a probability distribution in terms of another variable related by a constant to the original; i.e., express $p(R)$ as $p(aR)$ where a is a constant. Since the probability at any point is independent of the variable we have

$$p(R)dR = p(aR)d(aR)$$

Hence $p(aR) = \dfrac{1}{a}p(R)$

i.e., $p(x) = \sqrt{2\psi_0}\, p(R)$
$= 2x \exp.[-(x^2+y^2)]\, I_0(2xy)$

Since $p(x)$ is dependent on y we can define any particular distribution as

$$p(x, y) = 2x \exp.[-(x^2+y^2)]\, I_0(2xy)$$

thus when no signal is present, $y = 0$ and we have

$$p(x, 0) = 2x \exp.[-x^2]$$

i.e., the well-known Rayleigh distribution.

Fig. 1 shows the form of $p(x, y)$ for various values of y. [These curves have been abstracted from Rice's paper.] The mean is given by

$$\frac{\int_0^\infty x p(x, y)dx}{\int_0^\infty p(x, y)dx}$$

and the mean square by $\dfrac{\int_0^\infty x^2 p(x, y)dx}{\int_0^\infty p(x, y)dx}$

*Contrast: Throughout this paper the term contrast is used in the sense of a bias; i.e., the input has to reach a predetermined level before its effect is registered. This is similar to the photographer's use of the term, in that the difference between the high and low amplitudes is exaggerated.

†A full discussion of detectability criteria can be found in reference 2.

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All these integrals may be evaluated. But when the limits are not zero and infinity the integration can be carried out only by numerical methods so long as $y > 0$.

When contrast in the form of a bias voltage or current is applied, all amplitudes below a given value A are not displayed. This of course alters the mean and standard deviation of the remaining distribution and the limits of integration are now from A to infinity. In the Appendix the new mean and standard deviations for the Rayleigh distribution are derived, but the mean of the distribution including signal requires numerical calculation. Fortunately the integral $\int_A^\infty p(x,y)dx$ has been tabulated over quite a wide range and a small approximation enables the integral $\int_A^\infty xp(x,y)dx$ to be calculated, viz.

$$\int_A^\infty xp(x,y)dx = \sum_{n=0}^{n=\infty}\left\{A + (2n+1)\frac{\Delta}{2}\right\}\int_{A+n\Delta}^{A+(n+1)\Delta} p(x,y)dx$$

$$= \sum_{n=0}^{n=\infty}\left\{A + \frac{(2n+1)\Delta}{2}\right\}\left\{\int_0^{A+(n+1)\Delta} p(x,y)dx - \int_0^{A+n\Delta} p(x,y)dx\right\}$$

The accuracy of this method obviously depends on the interval Δ which is of course limited by the original tabulation of $\int_0^A p(x,y)dx$. (The interval of the table available was 0·1 over most of the range.) A check was obtained by using this method to evaluate the integral from zero to infinity since the answer to this can be obtained analytically, and the resulting accuracy was found to be about 0·1%.

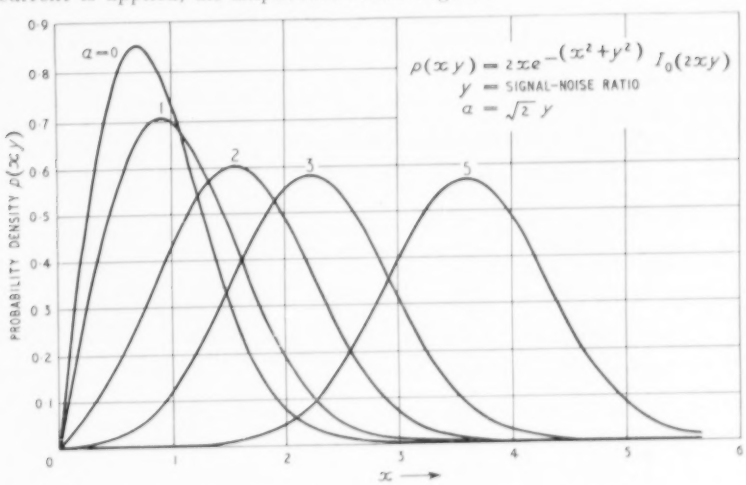


Fig. 1. *Probability density of the envelope (from Ref. 3).*

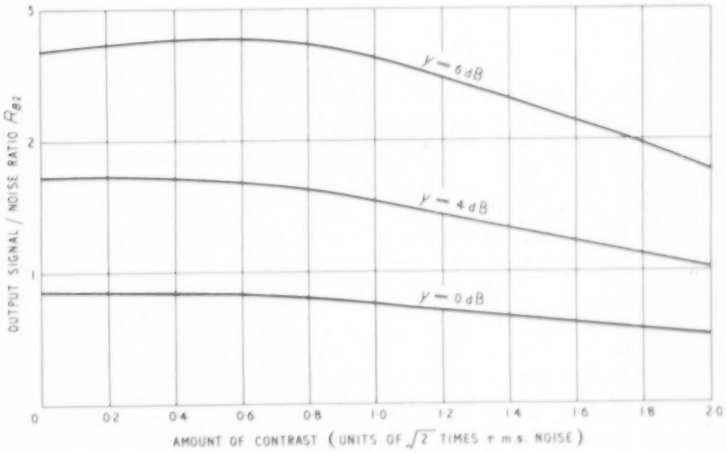


Fig. 2. *Effect of contrast on R_{82}.*

3. Results

It was not thought worth while to work out more than one or two examples to determine the trend of the result and these are shown graphically in Fig. 2 for signal/noise ratios of 0, 4, and 6 dB ($y = 0$, 1·5 and 2). The scale of the abscissa is in units of x, i.e., the point 1 is where $x = \dfrac{R}{\sqrt{2\psi_0}} = 1$. This means that at $x = 1$ the

amplitude R has to exceed $\sqrt{2\rho_0}$ before it is displayed; i.e., R must exceed $\sqrt{2}$ times the r.m.s. of the noise. Since less than 2% of the noise peaks exceed $2\sqrt{2}$ times the r.m.s. then the normal range of contrast is below $x = 2$.

4. Discussion and Conclusions

The dependence of the numerical value of the criterion R_{B2} on contrast is a function of the signal/noise ratio. Increase of contrast causes a reduction of the standard deviation when no signal is present, and an increase in the mean both with and without signal present. Consequently, unless this increase in means has a significant effect on the difference between them, the decrease in standard deviation would cause an increase in the value of R_{B2}. Hence for large signal/noise ratios, where the mean when signal is present is relatively high, the value of R_{B2} increases at first with contrast. Further increase in contrast, however, causes a reduction in the difference of means at a greater rate than the fall of the standard deviation, thus causing a reduction in the values of R_{B2}. For small signal/noise ratios the difference in means decreases sufficiently to counterbalance the fall in standard deviation even with small amounts of contrast.

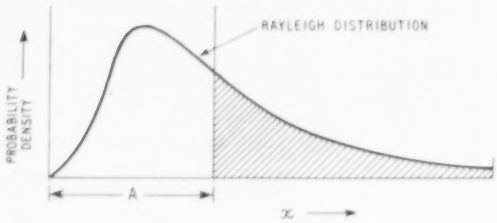
The general tendency, for the signal/noise ratios considered (0–6 dB), is that reasonable amounts of contrast have little effect on R_{B2} but large amounts cause a considerable reduction of its value. This would have been the predicted result of the effect of contrast on the probability of detection using the simplified distributions assumed in reference 4. Although this agreement must be regarded as only qualitative (since probabilities of detection for real distributions have not yet been calculated), yet it adds support for the use of the criterion R_{B2} as a measure of the relative efficiency of detector circuits for any given display medium. This is of importance, since criteria like R_{B2} are usually much more easily calculated than probabilities of detection. Reference 5 has already indicated that R_{B2} may well be a reliable criterion for the relative effectiveness of detection using linear and square-law detector circuits with A-scan display; square-law response is, of course, another kind of contrast, since the larger amplitudes are accentuated relative to the smaller ones.

Acknowledgments

The author has much pleasure in acknowledging the help given by Professor D. G. Tucker in the preparation of the manuscript. The paper is published by permission of the Admiralty.

APPENDIX

To obtain the mean and standard deviation of a contrasted Rayleigh Distribution



$$\text{Shaded area} = \int_A^\infty 2x\, e^{-x^2}\, dx = e^{-A^2}$$

$$\text{1st moment} = \int_A^\infty 2x^2\, e^{-x^2}\, dx = -\int_A^\infty x\, d(e^{-x^2})$$

$$= -\Big[xe^{-x^2}\Big]_A^\infty + \int_A^\infty e^{-x^2}\, dx$$

$$= Ae^{-A^2} + \frac{\sqrt{\pi}}{2}\,[1 - \text{erf. } (A)]$$

$$\therefore \text{Mean} = A + \frac{\sqrt{\pi}}{2}\, e^{A^2}\,[1 - \text{erf. } (A)]$$

$$\text{2nd moment} = \int_A^\infty 2x^2\, e^{-x^2}\, dx = -\int_A^\infty x^2\, d(e^{-x^2})$$

$$= -\Big[x^2\, e^{-x^2}\Big]_A^\infty + \int_A^\infty x\, e^{-x^2}\, dx$$

$$= A^2 e^{-A^2} + e^{-A^2}$$

$$\therefore \text{Mean Square} = 1 + A^2$$

$$\text{Standard Deviation} = \{\text{Mean Square} - (\text{Mean})^2\}^{\frac{1}{2}}.$$

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THE IMPEDANCE CONCEPT

Part 2—Quartic and Quintic Characteristic Equations: Maximum Feedback

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SUMMARY.—In Part 1, the fundamental properties of impedances, and of the 'p-world' to which these impedances belong, were discussed. A general condition for an algebraic equation to be free from roots with positive real parts was obtained; a network whose characteristic equation has this property is stable. Here we consider in more detail the effect on stability and damping of adding terms in p^4 and p^5 to a given cubic in p. The addition of extra 'step-circuit' elements to a three-stage RC amplifier, in order to improve feedback, is then discussed. Only the parameters associated with the additional elements are under our control, and a graphical method of finding, roughly, values of these parameters suitable for obtaining a given amount of feedback is explained. Only elementary algebra is involved, including the condition for a quadratic equation to have real roots, provided that the characteristic equation of the amplifier, with the additional elements is, at worst, quintic. The procedure can be extended to characteristic equations of higher degree, in the manner indicated by a numerical example where the degree is 7.

LIST OF PRINCIPAL SYMBOLS

$a_0, a_1, a_2 \ldots a_n$	Coefficients of the polynomial P_n.
A	Transfer function of amplifier without step-circuit [equation (3)].
A'	Transfer function of amplifier with step-circuit [equation (7)].
b	Parameter associated with the step-circuit [equation (7)].
b_1, b_2, b_3, b_4	Parameters defined by equations (9) associated with the characteristic equation of a feedback amplifier with three RC stages and a step-circuit.
B	Parameter associated with the step-circuit [equation (7)].
E_n	Real part of the polynomial P_n when p is replaced by $j\omega$.
k	Parameter associated with the step-circuit [see equation (7)].
n	Degree of the polynomial P_n.
O_n	Imaginary part of the polynomial P_n when p is replaced by $j\omega$.
p	Heaviside operator, in terms of which impedances are expressed. In this article p can often be replaced by $j\omega$.
P_n	A polynomial of degree n in p.
T_1, T_2, T_3	Time constants of three RC amplifier stages [equation (3)].
β	Transfer function of the feedback path [equation (4)].
λ	Feedback [defined precisely just before equation (4)].
Λ_0	Value of λ which makes E_7 have minimum value zero [Section 6].
Λ_1	Value of λ which makes O_7 have minimum value zero [Section 6].
ω	See definition of p.
$\omega_1, \omega_2 \ldots$	Zeros of E_n (in ascending order).
$\omega_{m1}, \omega_{m2} \ldots$	Turning values of E_n (in ascending order).
$\Omega_1, \Omega_2 \ldots$	Zeros of O_n (in ascending order).

1. Introduction

In Part 1 we showed how useful information on circuit behaviour could be obtained within the 'p-world', p being a variable closely associated with time differentiation which can be manipulated algebraically. A general condition for a polynomial P_n of degree n in p to be free from zeros with positive real parts was obtained, and applied to the case of a three-stage RC feedback amplifier. Here our main objective is to determine the effect of adding extra 'step-circuit' elements on the gain and the maximum feedback obtainable. So we first derive the characteristic equation as modified by the presence of the extra elements. The parameters associated with the extra elements are in fact involved in all the coefficients of the characteristic equation, but for the purposes of discussion and understanding it will be easier initially to consider how the roots of a given cubic equation in p, namely

$$P_3 \equiv a_0 + a_1 p + a_2 p^2 + a_3 p^3 = 0 \quad \ldots \quad (1)$$

are affected by the addition of terms $a_4 p^4$ and $a_5 p^5$, so that we compare the roots of (1) with those of

$$P_5 \equiv a_0 + a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4 + a_5 p^5 = 0 \quad (2)$$

when a_0, a_1, a_2, and a_3 are regarded as fixed and out of our control but a_4 and a_5 are regarded as variable and under our control.

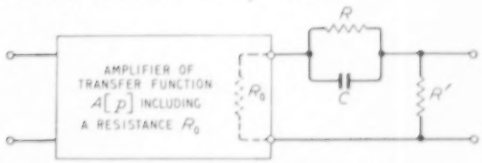

Fig. 8. *Three-stage amplifier with added step circuit elements to improve feedback. (This is repeated from Part 1 for easy reference.)*

2. Characteristic Equation of a Three-Stage *RC* Amplifier with Added Elements to improve Feedback

We shall suppose that, before the additional 'step-circuit' elements indicated in Fig. 8 are introduced, our three-stage RC amplifier has the transfer function

$$A = A_0/[(1 + T_1 p)(1 + T_2 p)(1 + T_3 p)] \quad (3)$$

where A_0 is a constant, and the time constants T_1, T_2 and T_3 are determined by the parameters of the three valves and are not under our control except in so far as some trading of bandwidth for gain is possible. R_0 is taken into account in the determination of T_1, T_2 and T_3. We shall also suppose that negative feedback is introduced, so that the transfer function $\beta[p]$ of the feedback path is a mere negative constant $-\lambda/A_0$. The characteristic equation before the additional elements are introduced is then

$$1 - A\beta = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

which reduces to

$$(1 + \lambda) + (T_1 + T_2 + T_3)p + (T_2T_3 + T_3T_1 + T_1T_2)p^2 + T_1T_2T_3p^3 = 0 \quad (5)$$

and in Part 1 we found that the condition for stability was

$$\lambda < (T_2/T_3) + (T_3/T_2) + (T_3/T_1) + (T_1/T_3) + (T_1/T_2) + (T_2/T_1) + 2 \quad \ldots \quad (6)$$

The effect of the additional elements is to replace the original forward gain A by A' where

$$A' = kA\,\frac{1 + bp}{1 + Bp} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

k being a constant less than unity and b, B being positive constants such that $b > B$ and $k < B/b$, since the additional elements are passive, so that their presence cannot increase the gain at low or high frequencies. k, b and B are determined in terms of the valve constants of the original amplifier, the position at which the additional elements are attached, and the magnitudes of these additional elements. The gain is thus reduced in the ratio k:1 (which we can control by adjusting the point of attachment of the additional elements) at low frequencies, but is reduced in the higher ratio kb/B at high frequencies. In the characteristic equation (4) we have to replace A by A', so that (4) reduces to

$$(1 + \lambda k) + (b_1 + \lambda kb)p + b_2p^2 + b_3p^3 + b_4p^4 = 0 \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

where

$$\left.\begin{aligned} b_1 &= (T_1 + T_2 + T_3) + B \\ b_2 &= (T_2T_3 + T_3T_1 + T_1T_2) + B(T_1 + T_2 + T_3) \\ b_3 &= T_1T_2T_3 + B(T_2T_3 + T_3T_1 + T_1T_2) \\ b_4 &= T_1T_2T_3B \end{aligned}\right\} \quad (9)$$

(8) is thus a quartic in p, and the coefficients of the terms in 1, p, p^2 and p^3, though not identical with the corresponding coefficients of (5), are closely related to them, especially when b, B and λ are small and k is near 1. It will be easier to understand the significance of (9) if we first compare the roots of (1) and (2).

3. The Effect of Adding Quartic and Quintic Terms to a Given Cubic in p

At first, suppose that, in (2), a_5 is zero while a_4 is gradually increased from zero. It is shown in Part 1 that (2) must have roots with positive real parts if

$$a_5 = 0,\ a_4 > a_2^2/4a_0 \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

since then the real part E_4 of P_4 when p is replaced by $j\omega$, namely

$$E_4 = a_0 - a_2\omega^2 + a_4\omega^4, \quad \ldots \quad \ldots \quad (11)$$

would have complex zeros. We are thus not at liberty to increase a_4 indefinitely. Again, the zeros of the imaginary part of O_4 of P_4 when p is replaced by $j\omega$, namely

$$O_4 = \omega(a_1 - a_3\omega^2) \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

occur when $\omega = 0$ and $\omega^2 = (a_1/a_3)$. If P_5 (with $a_5 = 0$) is free from zeros with positive real parts, it is shown in Part 1 that this last value of ω^2 must separate the two values of ω^2 which make E_4 zero when a_4 is small enough for these values of ω^2 to be real. An alternative way of expressing this is to say that E_4 in (11) must be negative when $\omega^2 = (a_1/a_3)$.

Now if

$$a_1/a_3 < a_0/a_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

the value of E_4 is necessarily positive when $\omega^2 = a_1/a_3$ since $(a_0 - a_2\omega^2)$ is then positive. But (13) happens also to be the condition that (1) shall have a root with a positive real part. It follows that if the characteristic equation (1) is associated with instability, the addition of a quartic term cannot correct the instability.

When (13) is not true, (2) with $a_4 = a_5 = 0$ can be regarded as the characteristic equation of a stable system; if $a_5 = 0$ but a_4 is varied, the zero Ω_1 of O_4 remains constant at $(a_1/a_3)^{\frac{1}{2}}$ while the zeros ω_1, ω_2 ($\omega_1 < \omega_2$) of E_4 are given by

$$1/\omega_1^2 = \left[a_2 + \left\{a_2^2 - 4a_0a_4\right\}^{\frac{1}{2}}\right]/2a_0;$$
$$1/\omega_2^2 = \left[a_2 - \left\{a_2^2 - 4a_0a_4\right\}^{\frac{1}{2}}\right]/2a_0 \quad \ldots \quad (14)$$

When a_4 is zero, $\omega_1^2 = a_0/a_2$, ω_1 is less than Ω_1, and ω_2 is infinite. As a_4 increases, ω_1 increases and ω_2 decreases until, just before (10) is violated, ω_1^2 and ω_2^2 are both near $2a_0/a_2$ but $\omega_1^2 < 2a_0/a_2 < \omega_2^2$. If therefore

$$2a_0/a_2 > a_1/a_3 > a_0/a_2 \quad \ldots \quad \ldots \quad \ldots \quad (15a)$$

the effect of increasing a_4 is to make ω_1 approach Ω_1 from below, whereas if

$$a_1/a_3 > 2a_0/a_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15b)$$

the effect of increasing a_4 is to make ω_2 approach Ω_1 from above. The important point is that, in either case, increasing the quartic term tends to reduce the difference between Ω_1 and one or other of the zeros ω_1, ω_2 of E_4, and this is associated with a reduction in the effective damping rate of the system whose characteristic equation is (2) with $a_5 = 0$.

Now let us consider what happens when a_5 is not zero. There is no possibility of avoiding roots with positive real parts unless

$$a_2^2 > 4a_0a_4\ ;\ a_3^2 > 4a_1a_5 \quad \ldots \quad \ldots \quad (16)$$

(so that we are not at liberty to increase a_5 indefinitely) because for stability we now have (as shown in Part 1) that the zeros of

$$E_5 = a_0 - a_2\omega^2 + a_4\omega^4 \quad \ldots \quad \ldots \quad (17)$$

must be real and separate those of

$$O_5 = \omega[a_1 - a_3\omega^2 + a_5\omega^4] \quad \ldots \quad \ldots \quad (18)$$

Since E_5 is the same as E_4 in (11), its zeros ω_1, ω_2 are given by (14), while the zeros Ω_1, Ω_2 of O_5 are given by

$$1/\Omega_1^2 = \left[a_3 + \left\{a_3^2 - 4a_1a_5\right\}^{\frac{1}{2}}\right]/2a_1;$$

$$1/\Omega_2^2 = \left[a_3 - \left\{a_3^2 - 4a_1a_5\right\}^{\frac{1}{2}}\right]/2a_1 \quad (19)$$

For stability we require

$$\omega_1 < \Omega_1 < \omega_2 < \Omega_2 \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

and we note that

ω_1 increases with a_4 but is independent of a_5

Ω_1 increases with a_5 but is independent of a_4

ω_2 decreases as a_4 increases but is independent of a_5

Ω_2 decreases as a_5 increases but is independent of a_4

By careful consideration of the behaviour of (14) and (19) when a_4 and a_5 vary, it can be shown that if (13) is true, it is not possible to find a_4 and a_5 so that (2) is free from roots with positive real parts. If, however, (13) is untrue, so that (1) is free from roots with positive real parts, then given any value of a_4 less than $a_2^2/4a_0$ [so that (17) has real roots] it is always possible to find a range of values of a_5 such that (2) is free from roots with positive real parts, and given any value of a_5 less than $a_3^2/4a_1$ [so that (18) has real roots] it is always possible to find a range of values of a_4 such that (2) is free from roots with positive real parts. This can be best illustrated by means of two examples, in the first of which (1) has a root with positive real part while in the second (1) has no such root. For these examples, we shall require the well-known condition for the equation (2) to have a purely imaginary root, which is

$$(a_0a_5 - a_1a_4)^2 = (a_2a_5 - a_3a_4)\,(a_0a_3 - a_1a_2) \quad \ldots \quad \ldots \quad (21)$$

Consider then the equation

$$1 + p + 4p^2 + 5p^3 + a_4p^4 + a_5p^5 = 0 \quad (22)$$

Here the cubic obtained by equating a_4 and a_5 to zero has a root with positive real part.

(21) reduces to

$$(a_5 - a_4)^2 = 4a_5 - 5a_4 \quad \ldots \quad \ldots \quad (23)$$

and if a_4 is plotted against a_5 to satisfy (23), we obtain the parabola in Fig. 9. The number of roots of (22) with positive real parts in (22) cannot change when we vary (a_4, a_5) unless this variation causes the parabola (23) to be crossed. If $a_5 = 0$ while a_4 is very small, (22) will have roots very similar to those of the cubic obtained by equating a_4 and a_5 to zero, together with a large negative root near $-5/a_4$. Hence the exterior of the parabola is all a region in which (22) has one pair of complex roots with positive real parts. If (a_4, a_5) is inside the parabola it will be found that (22) has two pairs of complex roots with positive real parts.

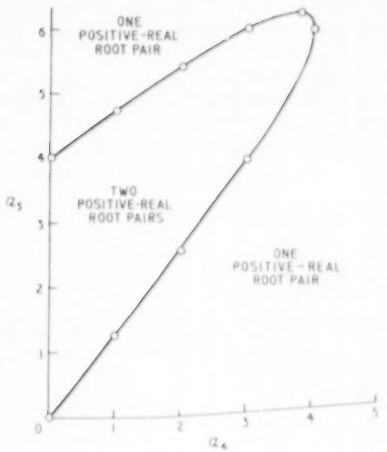

Fig. 9. *a_5 plotted against a_4 to satisfy (23). Nature of roots of (22).*

Now suppose that the coefficients of p^2 and p^3 are exchanged in (22) so that the equation being considered is

$$1 + p + 5p^2 + 4p^3 + a_4p^4 + a_5p^5 = 0 \quad (24)$$

If $a_4 = a_5 = 0$ we now have a cubic free from roots with positive real parts. (21) becomes

$$(a_5 - a_4)^2 = 4a_4 - 5a_5 \quad \ldots \quad \ldots \quad (25)$$

so that the locus of points for which (24) has purely imaginary roots is obtained from Fig. 9 by exchanging a_4 and a_5, and is as in Fig. 10. In this case we find that if (a_4, a_5) is within the part of the parabola (25) which is in the first quadrant, (24) is free from roots with positive real parts, but if (a_4, a_5) is outside the parabola and in the first quadrant, there is one pair of roots with positive real parts. In Fig. 10, we notice

(i) As a_5 increases, the range of permissible values of a_4 decreases; this range never includes zero.

(ii) If $4 < a_4 < 6{\cdot}25$, the range of permissible values of a_5 does not include zero. This means that the quartic characteristic equation associated with an unstable system obtained from (24) by putting $a_5 = 0$ and giving a_4 a value between 4 and 6·25 can be 'corrected' by the addition of a quintic term whose coefficient is between certain limits which approach each other as the coefficient a_4 approaches the maximum permissible value $a_2^2/4a_0 = 6{\cdot}25$.

(iii) No 'correction' is possible if either a_4 exceeds $a_2^2/4a_0$ or a_5 exceeds $a_3^2/4a_1$.

The peculiar result (ii) does not always occur. The general condition for it to occur is derived from (21) by putting $a_5 = 0$. If the non-zero value $C = a_3(a_1a_2 - a_0a_3)/a_1^2$ of a_4 thus obtained is such that the slope of the tangent at $(C, 0)$ to the parabola (21) is positive, we shall have a similar situation to that of Fig. 10, but not in the contrary case. It thus appears that the situation of Fig. 10 will occur when

$$a_0a_3 < a_1a_2 < 2a_0a_3$$

since Fig. 10 is not applicable at all unless (13) is untrue.

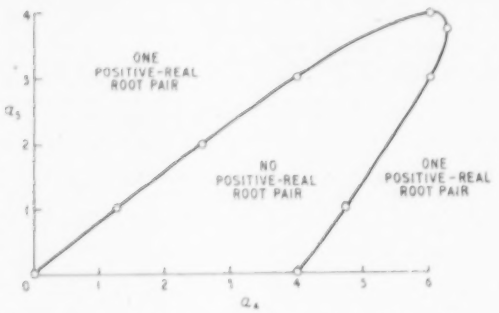

Fig. 10. *a_5 plotted against a_4 to satisfy (25). Nature of roots of (24).*

4. Nature of the Roots of the Characteristic Equation of the Three-Stage *RC* Amplifier with Added Elements

We now have to apply the above considerations to (8). The zero Ω_1 of (12) is given by

$$\Omega_1^2 = (b_1 + \lambda kb)/b_3 \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

while those of (11) are given by

$$(1 + \lambda k) - b_2\omega^2 + b_4\omega^4 = 0 \quad \ldots \quad (27)$$

The condition that (26) shall separate (27), so that (8) has no roots with positive real parts can in this case be written down; it reduces to

$$b^2b_4\lambda^2k^2 - \lambda k\{b_1b_2b_3 - 2bb_1b_4 - b_3^2\} < \{b_1b_2b_3 - b_3^2 - b_4b_1^2\} \quad \ldots \quad (28)$$

The term independent of λ in (28) is positive since when $\lambda = 0$ (8) has four real negative roots, $-T_1$, $-T_2$, $-T_3$ and $-B$, in p. For very large λ the first term of (28) prevails, so that there is a limit to the feedback obtainable. But although in any particular case (28) can be solved to obtain the maximum feedback, it is not easy to deduce from (28) how to adjust k, b and B so as to increase the feedback. If however we apply (13) to (8) we see that there cannot possibly be stability unless the cubic equation formed from (8) by omitting the last term is free from roots with positive real parts, so that

$$b_1b_2 - b_3 > \lambda k\{b_3 - bb_2\} \quad \ldots \quad \ldots \quad (29)$$

Similarly, if we put $p = 1/q$ in (8), we should obtain a quartic equation for q which must be free from roots with positive real parts (the real parts of p and q have the same sign). Applying (13) to this q-equation, we find that there cannot be stability unless

$$(b_1 + \lambda kb)q^3 + b_2q^2 + b_3q + b_4 = 0 \quad \ldots \quad (30)$$

is free from roots with positive real parts; this requires

$$b_2b_3 - b_1b_4 > \lambda kbb_4 \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

(29) may be true for all positive λk, but (31) will always set an upper limit to λk. If (29) and (31) are both satisfied, (8) may be associated with a stable system if B is sufficiently small. Thus (29) and (31) indicate the order of magnitude of the maximum feedback obtainable. It is useless to proceed further, whether by examining (28) or by examining the behaviour of (26) and (27) as functions of λ, unless B, b and k can be adjusted so that the required value of λ can be substantially exceeded without violating (29) or (31).

5. Choice of the Parameters under our Control to obtain Improved Feedback

Let us now consider a specific problem: if $T_1 = 100$, $T_2 = 10$ and $T_3 = 1$, the maximum feedback obtainable from the circuit without additional elements is deducible from (6) and is 122·21. From the nature and arrangement of the added elements, there must inevitably be some loss of gain (i.e., k must be less than 1) but we do not want the loss of gain to be excessive, so we can perhaps take a reasonable value of k to be 0·2; what values of b, B, if any, are then suitable if we wish to increase the maximum feedback obtainable to 1,500 with $k = 0{\cdot}2$? In this case (8) reduces to

$$(1 + \lambda k) + (111 + B + \lambda kb)p + (1110 + 111B)p^2 + (1000 + 1110B)p^3 + 1000Bp^4 = 0 \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

(29) and (31) become respectively

$$\lambda k < \frac{122{\cdot}21 + 12{\cdot}321B - 0{\cdot}111B^2}{1 - 1{\cdot}11\ (b - B) - 0{\cdot}111bB} \quad \ldots \quad (33)$$

and

$$\lambda k < \frac{999 + 1342{\cdot}1B + 123{\cdot}21B^2}{b} \quad \ldots \quad (34)$$

Now we saw at the end of Section 4 that (29) and (31) should be satisfied for a value of λ considerably in excess of the required value. In (33) and (34), therefore, we put 1800 instead of 1500 for λ; k is 0·2. We thus obtain from (33)

$$b > \frac{0{\cdot}5951 + 0{\cdot}9692B + 0{\cdot}000278B^2}{1 + 0{\cdot}1B} \quad (35)$$

and from (34)

$$b < 2{\cdot}775 + 3{\cdot}7281B + 0{\cdot}34225B^2 \quad \ldots \quad (36)$$

We also know that

$$b > B > kb \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

and that the roots of (27) must be real. The latter fact in our case gives

$$(1110 + 111B)^2 > 4000B\,(1 + \lambda k) \quad \ldots \quad (38)$$

(38), like (33) and (34), should be satisfied for a value of λ considerably greater than the required value of 1,500, since satisfying (38) for $\lambda = 1{,}500$ only ensures that the roots of (27) are real. If these roots are close together it may be very difficult to ensure that they are separated by (26). In (38) therefore, we put $\lambda = 1{,}800$ and $k = 0{\cdot}2$; we deduce that B is not between 1·04 and 96·16.

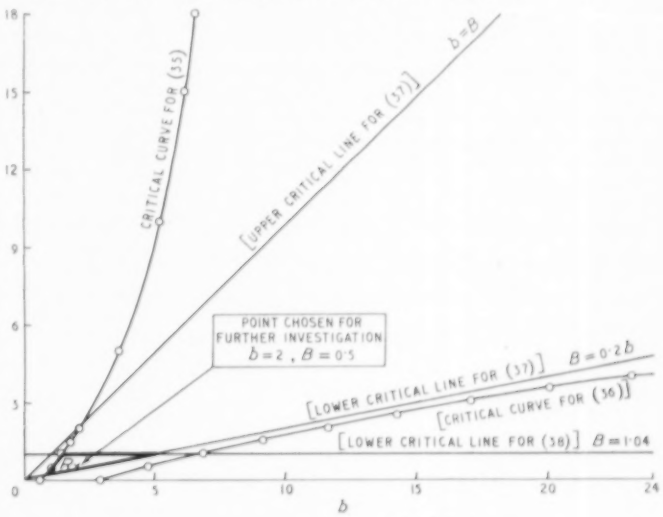

Fig. 11. Regions of the (b, B) plane in relation to (35), (36), (37) and (38).

In Fig. 11, the positive quadrant of the (b, B) plane is divided into regions according to whether (i) $(b - B)$ is positive or negative, (ii) (35) is true or false, (iii) (36) is true or false, and (iv) the roots of (27) are real or complex; i.e., (38) is true or false. It is thus clear that a possible region in which suitable points (b, B) are likely to be found is the region labelled R in Fig. 11, bounded by $B = 0{\cdot}2b$ below, by $b = (0{\cdot}5951 + 0{\cdot}9692B + 0{\cdot}000278B^2)/(1 + 0{\cdot}1B)$ on the left, and by $B = 1{\cdot}04$ above. A suitable point within this region would appear to be $b = 2$, $B = 0{\cdot}5$, and we shall arbitrarily choose this for further examination. Practical considerations might easily cause us to be particularly interested in some other part of the region R, but the point we wish to emphasize here is that it is useless to attempt to look outside the region R, or possibly the region S described next.

The region S (which cannot be included in Fig. 11 because a scale which allowed us to include it would make the region R too small) is bounded on the left by the line $b = B$, and bounded below by $B = 96{\cdot}16$ for $96{\cdot}16 < b < 480{\cdot}8$ and by the line $B = 0{\cdot}2b$ for $B > 480{\cdot}8$, this line being always above the parabola along which the two sides of (36) are equal. A typical point well within this region is $b = 150$, $B = 120$. For $b = 2$, $B = 0{\cdot}5$, $k = 0{\cdot}2$, (32) reduces to

$$(1 + 0{\cdot}2\lambda) + \tfrac{4}{5}(111{\cdot}5 + 0{\cdot}4\lambda)p + 1165{\cdot}5p^2 + 1555p^3 + 500p^4 = 0 \quad \ldots \quad \ldots \quad (39)$$

In this case, (26) becomes

$$\Omega_1{}^2 = (111{\cdot}5 + 0{\cdot}4\lambda)/1555 = 0{\cdot}0717 + 2{\cdot}57235 \times 10^{-4}\lambda \quad \ldots \quad (40)$$

while (27) can be written

$$\lambda = -5 + 5827{\cdot}5\omega^2 - 2500\omega^4 \quad \ldots \quad (41)$$

From (41), it is clear that the sum of the values of ω^2 for a given λ is $5827{\cdot}5/2500 = 2{\cdot}331$. We now choose any value $\omega_1{}^2$ of ω^2, and deduce λ from (41). The other root of (41) for the same λ is $\omega_2{}^2 = 2{\cdot}331 - \omega_1{}^2$. The root $\Omega_1{}^2$ of (40) for the same λ is then deduced. In this way we obtain the corresponding sets of values of $\omega_1{}^2$, $\omega_2{}^2$, $\omega_3{}^2$ and λ shown in Table I

TABLE I

$\omega_1{}^2$	0·01	0·1	0·3	0·6	0·9	1
$\Omega_1{}^2$	0·0853	0·2139	0·4622	0·7383	0·8986	0·9264
$\omega_2{}^2$	2·321	2·231	2·031	1·731	1·431	1·331
λ	53·025	552·75	1518·24	2591·5	3214·7	3322·5

It is thus clear that stability breaks down because ω_1 approaches Ω_1 when λ is about 3,214, which is well in excess of the value 1,500 sought. For $b = 150$, $B = 120$, $k = 0{\cdot}2$, (32) reduces to

$$(1 + 0{\cdot}2\lambda) + (231 + 30\lambda)p + 14{,}430p^2 + 134{,}200p^3 + 120{,}000p^4 = 0 \quad \ldots \quad (42)$$

so that (26) becomes

$$\Omega_1{}^2 = (231 + 30\lambda)/134{,}200 = (1{\cdot}72131 \times 10^{-3}) + (2{\cdot}23547 \times 10^{-4}\lambda) \quad \ldots \quad (43)$$

while (27) can be written

$$\lambda = -5 + 72150\omega^2 - 600{,}000\omega^4 \quad \ldots \quad (44)$$

Proceeding as before $\omega_1{}^2 + \omega_2{}^2$ is now $72150/600{,}000 = 0{\cdot}12025$, while $\Omega_1{}^2$ is obtained from (43). We thus obtain the values ω_1, ω_2, Ω_1 and

λ shown in Table 2.

TABLE 2

ω_1^2	0·005	0·01	0·008	0·0078	0·0074
Ω_1^2	0·0779	0·1485	0·1210	0·1182	0·11261
ω_2^2	0·11525	0·11625	0·11225	0·11245	0·11285
λ	340·74	656·49	533·79	521·265	496·05

We thus have the disappointing result that stability is broken because Ω_1 becomes equal to ω_2 when λ is about 496 instead of the 1,500 sought. We now show that there are in fact no points (b, B) in the region S which give the required value of λ. ω_2^2 is necessarily less than the ratio of the coefficient of p^2 in (32) to the coefficient of p^4, and Ω_1^2 must be appreciably less than this ratio if there is to be any hope of stability for the required value of λ. We thus require, in addition to the conditions we have already obtained,

$$\frac{111 + B + \lambda kb}{1000 + 1110B} > \frac{1110 + 111B}{1000B} \quad \ldots \quad (45)$$

and since we also require $b > B$, it follows that

$$B < b < \left\{\frac{1110}{B} + 1232{\cdot}1 + 122{\cdot}21B\right\}/\lambda k \quad \ldots \quad (46)$$

If $\lambda = 1{,}500$ and $k = 0{\cdot}2$, the first and last members of (46) give, when B is positive

$$B^2 - 10{\cdot}08181B - 9{\cdot}08272 < 0 \quad \ldots \quad (47)$$

so that B must not exceed 10·914, whereas B must exceed 76·42 to satisfy (38) even with $\lambda = 1{,}500$ and $k = 0{\cdot}2$. This difficulty did not arise when b was 2 and B 0·5, because when stability was broken, ω_1 and not ω_2 became equal to Ω_1; (46) is easily satisfied by $B = 0{\cdot}5$, $b = 2$, $\lambda = 1{,}500$ and $k = 0{\cdot}2$.

In the foregoing discussion we admittedly gave particular values to T_1, T_2 and T_3 ab initio, but it is clear that the procedure discussed in no way depended on these particular values. The various curves involved in Fig. 11 might have different relative positions for other values of the T terms, and this might greatly affect the position and extent of the region R in which suitable points (b, B) were to be found. For certain values of the T terms and the required λ we might find that the inequalities corresponding to (38) and (46) were such that suitable points (b, B) could be found in the region corresponding to what we have called S as well as (or instead of) the region corresponding to what we have called R, but in any case the diagram corresponding to Fig. 11 shows us where to look for values of b and B such that high values of feedback can be obtained. We notice in (32) that when λ appears it is always associated with k; this means that any required value of λ can be obtained by sufficiently reducing k, but the procedure under discussion shows us the region(s) in which the appropriate values of b and B lie so that k can be reduced as little as possible. In the procedure discussed, only elementary algebra is involved, including the condition for a quadratic equation to have real roots. If the amplifier had had four stages instead of three, so that the equation corresponding to (8) was a quintic, we should have had an extra condition to consider, namely that the quadratic formed from the terms of odd degree in p would have to have real roots as well as the quadratic formed from the terms of even degree.

6. Extension to Characteristic Equations of Higher Degree

The procedure just discussed for finding b, B so that a given degree λ_0 of feedback is obtainable without instability for given k can be adapted to cases where the characteristic equation is of higher degree. We shall confine ourselves to the consideration of a particular case where the characteristic equation is of degree 7 when the additional elements are included. Thus the characteristic equation, corresponding to (4) cleared of fractions, will be taken as

$$\begin{aligned}&(1 + 100p)(1 + 50p)(1 + 20p)(1 + 10p)\\ &\quad\times(1 + 5p)(1 + 2p)(1 + Bp)\\ &\quad + k\lambda(1 + bp) = 0 \quad \ldots \quad \ldots \quad \ldots \quad (48)\end{aligned}$$

and we require to know in roughly what region of the (b, B) plane to look for the highest values of λ consistent with stability for a given k. Without the additional elements we should have $b = B = 0$ and $k = 1$. (48) reduces to

$$\begin{aligned}&P_7 \equiv (1 + k\lambda) + (187 + B + bk\lambda)p\\ &+ (10{,}970 + 187B)p^2 + (249{,}700 + 10{,}970B)p^3\\ &\quad + (2{,}357{,}000 + 249{,}700B)p^4\\ &\quad + (8{\cdot}8 + 2{\cdot}357B)10^6{\cdot}p^5 + (10 + 8{\cdot}8B)10^6{\cdot}p^6\\ &\quad + B{\cdot}10^7p^7 = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (49)\end{aligned}$$

We have no hope of stability unless the equations

$$\begin{aligned}&E_7 \equiv (1 + k\lambda) - (10{,}970 + 187B)\omega^2\\ &\quad + (2{,}357{,}000 + 249{,}700B)\omega^4\\ &\quad - (10 + 8{\cdot}8B)10^6\omega^6 = 0 \quad \ldots \quad \ldots \quad \ldots \quad (50)\end{aligned}$$

and

$$\begin{aligned}&O_7 \equiv (bk\lambda + 187 + B) - (249{,}700 +\\ &\quad 10{,}970B)\omega^2 + (8{\cdot}8 + 2{\cdot}357B)10^6\omega^4\\ &\quad - 10^7B\omega^6 = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (51)\end{aligned}$$

have all their roots real for the appropriate value of λ. Now consider E_7 in (50) as a function of ω^2. If for any given value λ_0 of λ, (50) has three real roots in ω^2, E_7 has the same value 0 for three different values ω_1^2, ω_2^2 and ω_3^2, of ω^2, and therefore E_7 has a negative minimum value for some value ω_{m1}^2 of ω^2 between ω_1^2 and ω_2^2, and a positive maximum value for some value ω_{m2}^2 of ω^2 between ω_2^2 and ω_3^2. It follows that $k\lambda_0$ cannot exceed the value kA_0 which makes the minimum of E_7 zero. If λ_0 is

slightly less than A_0, ω_1 and ω_2 will be close together and it will be necessary to adjust b in (51) so that O_7 has a root Ω_1^2 sufficiently near ω_{m1}^2 to ensure that $\omega_1^2 < \Omega_1^2 < \omega_2^2$ if there is to be any hope of stability with $\lambda = \lambda_0$. Similarly, $bk\lambda_0$ cannot exceed the value bkA_1 which makes the minimum value of O_7 zero.

First, let us consider the situation when $b = B = 0$ and $k = 1$; i.e., when the additional elements are not present. In this case O_7 is independent of λ, and the locus of points in the $(k\lambda, \omega^2)$ plane such that O_7 is zero consists of two vertical straight lines

$$\omega^2 = \omega_1^2 = 9{\cdot}26 \times 10^{-4}\ ;$$
$$\omega^2 = \omega_2^2 = 2{\cdot}295 \times 10^{-2} \quad \ldots \quad \ldots \quad (52)$$

[strictly speaking there is a third straight line which has gone to infinity because the ω^6-term in (51) is missing when $B = 0$]. The corresponding pairs of values of $k\lambda$ and ω^2 in Table 3 are sufficient to indicate the general nature of the locus of points in the $(k\lambda, \omega^2)$ plane, for $\omega^2 \leqslant 0{\cdot}005$, such that E_7 is zero:—

TABLE 3

$10^4\omega^2$	0	4	6	8	9·26	10
$k\lambda$	−1	3·01	4·74	6·27	7·17	7·62

$10^4\omega^2$	15	20	23·627	30	50
$k\lambda$	10·19	11·59	11·893 $= kA_0$	10·97	−3·83

For greater values of ω^2, $k\lambda$ remains negative until ω^2 reaches about 0·235. The other roots of E_7 when $k\lambda = 7{\cdot}15$ are thus approximately $\omega^2 = 33 \times 10^{-4}$ and $\omega^2 = 0{\cdot}24$ which are well separated by (52). Thus stability is broken when $b = B = 0$ because the least root of E_7 becomes equal to the least root of O_7, and this happens for a small value of ω^2 such that the ω^6 term in E_7 is of little importance. If we could keep B equal to zero and increase b, we might be able to raise the value of $k\lambda$ at which stability is broken from 7·15 to somewhat less than $kA_0 = 11{\cdot}89$, the value at which E_7 has a minimum value zero and $\omega_1^2 = \omega_2^2 = \omega_{m1}^2 = 23{\cdot}63 \times 10^{-4}$, but no adjustment of b can enable $k\lambda$ to exceed 11·89 without instability when $B = 0$.

The important question to consider next, therefore, is how kA_0 varies with B.

As the relevant values of ω are small when B is small, we can obtain kA_0 approximately by neglecting the ω^6 term in (50), in which case we have

$$kA_0 \approx \frac{(10{,}970 + 187B)^2}{4(2{,}357{,}000 + 249{,}700B)} - 1 = 0{\cdot}035011B + 2{\cdot}77723 + 8{\cdot}98698/\{1 + 0{\cdot}10594B\} \quad (53)$$

This expression is 11·764 when $B = 0$, which is very near the tabulated kA_0 of 11·893. It decreases as B increases to a minimum value 6·129 when $B = 61{\cdot}633$ and thereafter increases indefinitely, but does not regain the value 11·764 till $B = 247{\cdot}25$. This at first suggests that B should be large. Now b has to be greater than B, and if b and B are appreciable there are two dangers we have not previously had to consider, namely (i) the largest root of O_7, which is large when B is small, might become less than the largest root of E_7, and (ii) the value kA_1 of $k\lambda$ which makes O_7 have a minimum value zero may severely limit the permissible value of $k\lambda$. Now the highest roots of (50) and (51) are respectively approximately

$$\omega_3^2 = \frac{2{\cdot}357 + 0{\cdot}2497B}{10 + 8{\cdot}8B} \text{ and}$$
$$\Omega_3^2 = 0{\cdot}88 + \frac{0{\cdot}2357}{B} \quad \ldots \quad \ldots \quad (54)$$

but it is easily shown that these never became equal for positive B, so (i) need not be further considered. For (ii) we know that the minimum value of O_7 occurs for a relatively small value of ω^2, so that the ω^6 term contributes little, and we thus have, corresponding to (53),

$$kA_1 \approx \tfrac{1}{4}\frac{(249{,}700 + 10970B)^2}{(8{\cdot}8 + 2{\cdot}357B)10^6} - (187 + B)$$
$$= 11{\cdot}7642B + 462{\cdot}782 + \frac{1237{\cdot}89}{1 + 0{\cdot}26784B}$$
$$\ldots \quad \ldots \quad (55)$$

Now if $b = B = 247{\cdot}25$, so that kA_0 has regained its initial value 11·764, kA_1 derived from (55) is 13·710. An increase in B or b/B would reduce the value of kA_1 derived from (55), but would increase the value of kA_0 derived from (53). If we equate the value of kA_1 obtained from (55) for $b = B$ to the value of kA_0 obtained from (53), we find $b = B = 295$ approximately, while the corresponding common value of kA_0 and kA_1 is 13·38. This suggests that if $B = 295$ and $(b - B)$ is small, it should be possible to obtain values of $k\lambda$ in the region of 13·3. Unfortunately this possibility is not realized for we have to choose b so that the value of ω^2 which makes the right-hand side of (50) a maximum is approximately a root of (51). As $k\lambda$ is in the region of 13·3, this gives

$$\frac{13{\cdot}3b + B + 187}{249{,}700 + 10{,}970B} = \frac{10970 + 187B}{2(2{\cdot}357 + 0{\cdot}2497B)\cdot 10^6}$$

When $B = 295$, this requires $b = 77{\cdot}77$ which is hopelessly inconsistent with the condition $b > B$. We have, however, already seen that

values of $k\lambda$ up to about 11·7 can be obtained by making B as small as possible (so that kA_0, which decreases as B increases for small B, is little less than its value for B zero) and then choosing b so that the smallest zero Ω_1^2 of O_7 is equal to the value ω_{m1}^2 of ω^2 which makes E_7 a minimum. This gives us approximately

$$11{\cdot}7b + B + 187 \qquad 10{,}970 + 187B$$
$$249{,}700 + 10{,}970B = 2(2{\cdot}357 + 0{\cdot}2497B)\cdot 10^6$$

or

$$b = 0{\cdot}26562B + 9{\cdot}2903 + 230{\cdot}2418/\{B + 9{\cdot}4393\} \quad \ldots \quad \ldots \quad (56)$$

so that if $B = 0$, $b = 33{\cdot}68$. If $B = 10$, (56) gives $b = 23{\cdot}79$, but this is not quite correct since increasing B reduces kA_0 in (53), and therefore also reduces slightly the appropriate coefficient of b in (56). Thus in this case we can adjust the additional elements so that the maximum value of λk is increased by 60%, relative to the maximum value possible in the absence of additional elements. As before, we have to remember that k must not exceed B/b.

7. Conclusions

In Section 6 we have indicated approximately, but in a reasonably straightforward manner, optimum values of the parameters b and B associated with the additional 'step-circuit' elements which might be used for increasing the maximum feedback of a 6-stage RC amplifier, and the procedure could be carried out for any number of stages with quite sufficient accuracy for practical purposes.

If the particular example studied in Section 6 is typical, it appears to be very difficult to obtain, by means of the additional elements, a practically important increase in feedback without serious loss of gain for an amplifier with six stages, and presumably therefore for any larger number of stages. For a three-stage amplifier, however, the maximum feedback obtainable is readily calculated in the absence of the additional 'step-circuit' elements [equation (6)] and the presence of these additional elements appears to make very high values of feedback possible.

Given a quartic or quintic characteristic equation in which we are required to adjust the coefficients within given limits dictated by practical considerations, so that the equation shall be free from roots with positive real parts, the considerations which are most helpful in indicating the general range of values of the coefficients worth investigating are

(*a*) the equations formed from the terms of odd degree and terms of even degree respectively must have all their roots real.

(*b*) the cubic equation formed by the first four or the last four terms of the given quartic or quintic must be free from roots with positive real parts. For equations of higher degree (*a*) is still helpful, but consideration of the behaviour of the even-degree and odd-degree terms as functions of $\omega^2(p = j\omega)$ is a preferable alternative way of obtaining the same information.

Dr. J. THOMSON

A. J. Philpot is retiring in June from the directorship of the British Scientific Instrument Research Association. He will be succeeded by Dr. J. Thomson who has for some years been deputy director of physical research at the Admiralty. Previously he was professor of physics and electrical engineering at the Royal Naval College, Greenwich.

OBITUARY

Stanley Whitehead, M.A. (Oxon) D.Sc., M.I.E.E., F.Inst.P., died suddenly on 5th May. He was director of the British Electrical & Allied Industries Research Association to which post he was appointed in 1946 having been since 1939, first, assistant director and, later, deputy director.

STANDARD-FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Values for March 1956

Date 1956 March	Frequency deviation from nominal: parts in 10^8	
	MSF 60 kc/s 1429–1530 G.M.T.	Droitwich 200 kc/s 1030 G.M.T.
1	+0·3	−2
2	+0·3	−1
3	+0·3	−1
4	N.M.	−1
5	+0·3	−1
6	+0·3	0
7	+0·4	0
8	+0·4	0
9	+0·4	−1
10	+0·4	0
11	+0·4	−1
12	+0·4	0
13	+0·4	0
14	+0·4	+1
15	+0·4	0
16	+0·5	+1
17	N.M.	0
18	N.M.	N.M.
19	+0·4	−3
20	+0·5	+1
21	+0·5	+2
22	+0·5	+1
23	+0·5	+1
24	+0·5	+1
25	N.M.	N.M.
26	+0·4	−4
27	+0·4	−4
28	+0·5	−4
29	+0·5	−3
30	+0·5	−4
31	+0·5	−4

The values are based on astronomical data available on 1st April 1956

N.M. = Not Measured.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

RC Cathode-Follower Oscillators

SIR,—The appearance of a recent paper[1] has led us to look again at two papers[2,3] on the same subject which appeared in *Wireless Engineer*. In each of these papers an oscillator is described which is regarded as a combination of a passive *RC* network having a gain greater than unity, and a cathode follower. While it is not incorrect to regard such circuits in this way, there is the danger that so doing will lead to the following ideas, which, we submit, are fallacious:

(*a*) That the circuits are quite different from well-known and long established *RC* oscillator circuits.

(*b*) That due to the use of the valve as a cathode follower marked advantages are obtained, in particular that there will be a reduction in the influence of valve characteristics on waveform and frequency.

That these ideas are wrong may be shown simply by redrawing the circuits. In Fig. A we reproduce the circuit given in Fig. 8 of reference 3. Precisely the same circuit redrawn is shown in Fig. B, and a similar redrawing of the circuit given in Fig. 17 of reference 2 is shown in Fig. C. Apart from the position of the high-tension battery, which is of no theoretical significance, these redrawings show the valve as a conventional amplifier. It is now easier to regard the circuits as phase-shift oscillators, the *RC* networks providing the usual 180° of phase shift between anode and grid; and if the oscillators have any particular merits, it is, we suppose, because negative feedback is provided at frequencies both above and below the oscillation frequency, thus combining the properties of the two well-known networks shown in Fig. D.

The oscillator circuit given in reference 1 is of interest because by redrawing it is shown to be a well-known phase-shift oscillator. It is drawn by the author as shown in Fig. E and, if the cathode follower with a more than unity-gain network between cathode and grid aspect is to be stressed, the rearrangement shown in Fig. F is perhaps preferable. The identity of the circuit is revealed, however, by the redrawing shown in Fig. G, which shows the oscillator to be a single-valve Van der Pol.

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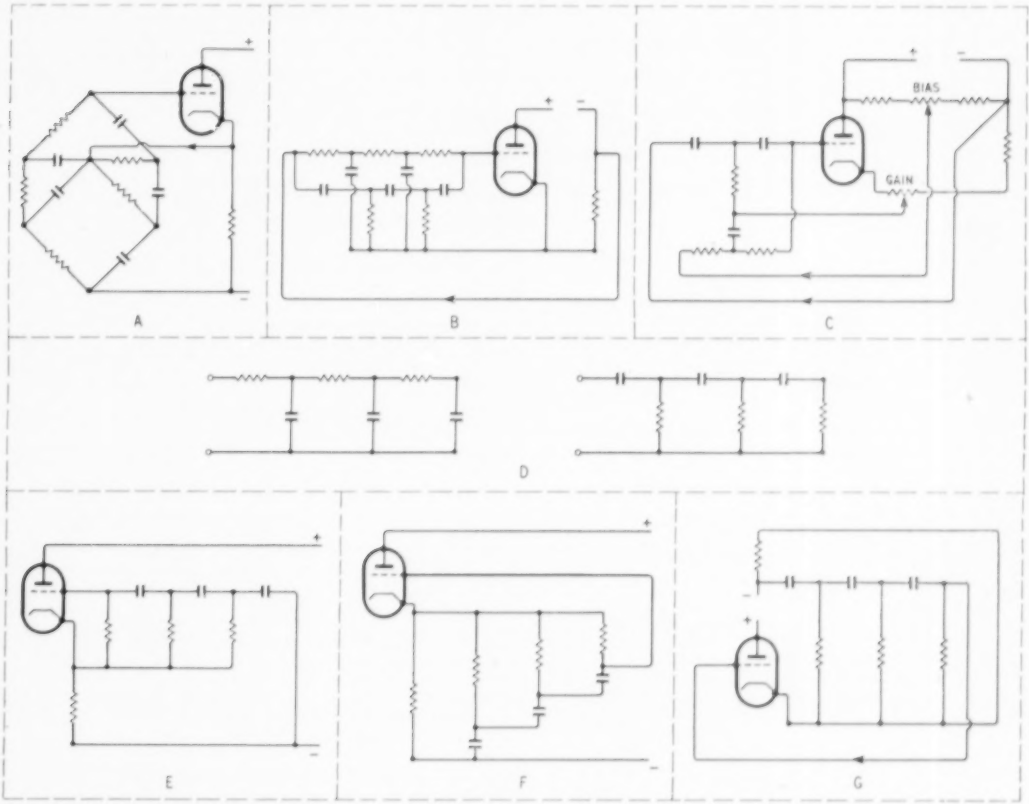
F. E. J. GIRLING, S. W. NOBLE.

R.R.E.,
Malvern, Worcs.
10th April 1956.



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3 W. Bacon and D. P. Salmon, "Resistance-Capacitance Networks with Over-Unity Gain", the same, p. 20.

NEW BOOKS

Electronic and Radio Engineering (4th Edition)

By F. E. TERMAN. Pp. 1078 + vii. McGraw-Hill Publishing Co. Ltd., 95 Farringdon Street, London, E.C.4. Price 71s. 6d.

The addition of "electronic" to the title of this well-known book is, perhaps, advantageous in affording a better indication of its contents. Normally, one would feel a change of title in such a book to be a bad thing but, in this case, the book has achieved the rare distinction of being better known by its author's name than by its title. Doubtless, this edition will still be called just 'Terman'!

In this edition, Professor Terman has been assisted by R. A. Helliwell, J. M. Pettit, D. A. Watkins and W. R. Rambo, all of Stanford University. There are 26 chapters and, after an initial chapter covering the elements of a system of radio communication, the book is divided into three sections. The first has four chapters covering circuit elements and circuit theory. A major part of this deals with transmission lines and waveguides, and the treatment of lumped circuits is rather brief.

The second section is on electronic engineering fundamentals and, with 16 chapters, forms the main part of the book. The aim here has been to deal with radio-like matters more generally than is normal in a radio book so that they can be applied to communication or non-communication apparatus more readily. Valves and electron optics are treated as well as amplifiers, feedback, oscillators, modulation, detection, wave-shaping, pulse techniques, microwave valves, power supplies and transistors.

The third section is the radio one, and has five chapters on propagation, aerials, transmitters and receivers, television and radar.

In its general style, this book closely follows the early editions and anyone used to these will feel at home with it. The treatment is not detailed and is rarely analytic. It is largely descriptive and formulae are usually quoted, not derived. W. T. C.

Principles of Radio

Wall charts, size 19 in. + 24 in. Educational Productions Ltd., East Ardsley, Wakefield, Yorks. Price 3s. per set of three.

These charts have been produced in collaboration with the E.M.I. Institute. They cover amplification, the tuned amplifier and the superheterodyne principle with a combination of diagram and very simple explanation.

Noise

By A. VAN DER ZIEL. Pp. 450 + xi. (Prentice-Hall Inc.) Chapman & Hall Ltd., 37 Essex Street, London, W.C.2. Price 60s.

This book covers most aspects of noise as it affects radio and electronic apparatus. The early chapters deal with thermal noise, Nyquist's and Schottky's theorems and the measurement of noise. There are individual chapters on noise in valves at low and at high frequencies, in mixing valves, in detectors and in feedback circuits.

Excess noise in semiconductors and valves is treated and there are chapters on statistical methods, Fourier analysis of fluctuating quantities and space-charge waves in electron beams. W. T. C.

Nomograms of Complex Hyperbolic Functions (2nd Edition)

By JØRGEN RYBNER. Published by Jul. Gjellerups Forlag, Copenhagen, Denmark. Price Kr. 44.00.

The first edition was reviewed in *Wireless Engineer* for April 1948, p. 129. In this new edition which, like the first, is in English and Danish, the notation for the complex argument of the hyperbolic functions has been changed from $g = b + ja$ to $\Gamma = A + jB$ in accordance with the recommendations of the International Telephone Consultative Committee. Four new nomograms have been added to extend the range of A to 3 instead of 2 nepers.

There are two other new nomograms for interaction loss and phase-shift. W. T. C.

International Scientific Radio Union

Proceedings of the XIth General Assembly:—

Vol. X, Part 5, Commission V. Radio Astronomy. Price 18s.

Vol. X, Part 6, Commission VI. Radio Waves and Circuits. Price 21s. 6d.

General Secretariat of the U.R.S.I., 42 Rue des Minimes, Brussels, Belgium.

Scattering and Diffraction of Radio Waves

By J. R. MENTZER. Pp. 134 + viii. Pergamon Press Ltd., 4–5 Fitzroy Square, London, S.W.1. Price 30s.

The author is Associate Professor of Electrical Engineering in the Pennsylvania State University. The book is a mathematical one and the analysis is concerned primarily with the solution of radar problems, not with scatter propagation.

Electrical Interference

By A. P. HALE, Graduate I.E.E. Pp. 122 + vii. Heywood & Co. Ltd. Tower House, Southampton Street, London, W.C.2. Price 15s.

Radio Amateurs' Handbook (33rd Edn.)

Pp. 620. American Radio Relay League Inc., West Hartford 7, Connecticut, U.S.A. Price $4.

Practical Radio Servicing

By WILLIAM MARCUS and ALEX LEVY. Pp. 565 + viii. McGraw-Hill Publishing Co. Ltd., 95 Farringdon Street, London, E.C.4. Price 49s.

Commercial Broadcasting in the British West Indies

Produced by Central Rediffusion Services Ltd. Pp. 91 + xii. Butterworths Publications Ltd., 88 Kingsway, London, W.C.2. Price 5s. (Postage 10d.).

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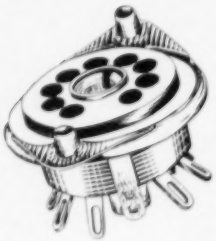

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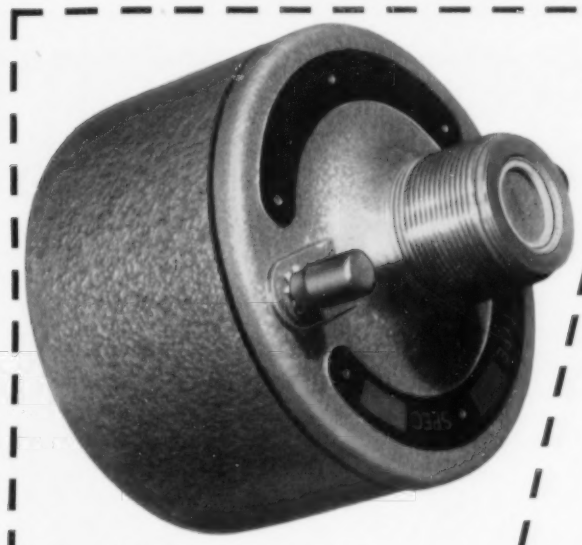

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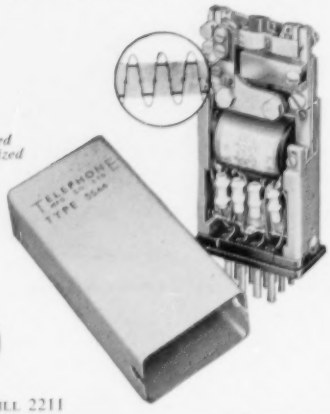

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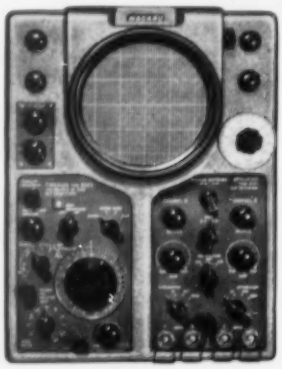

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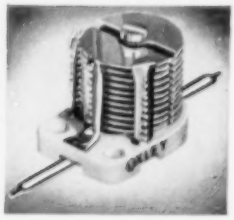

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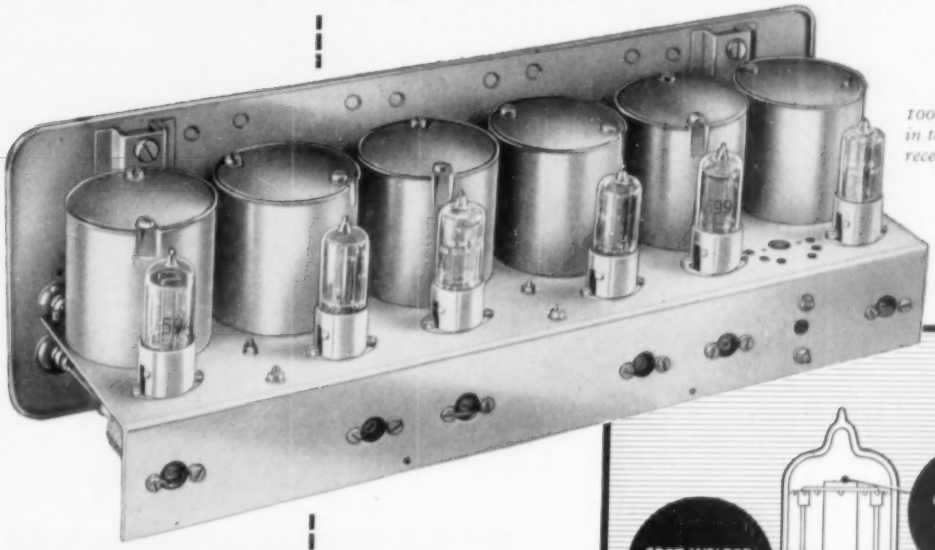

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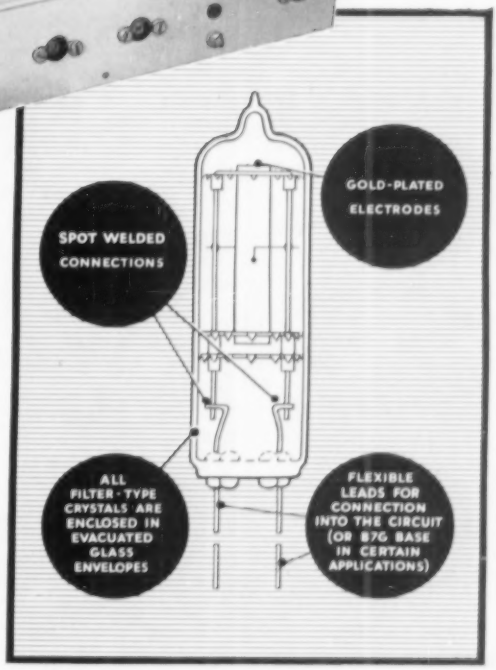
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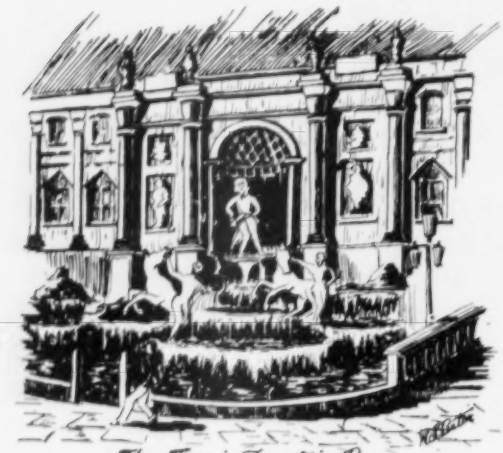

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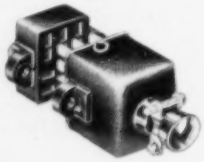

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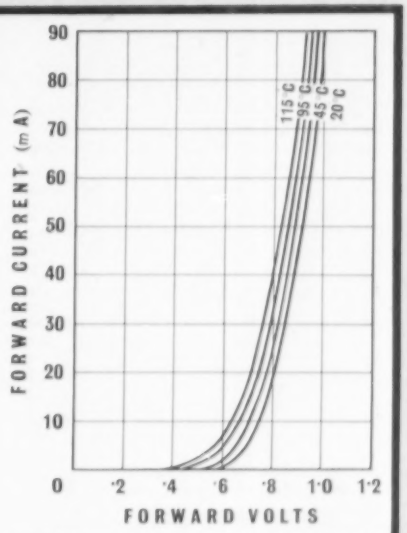

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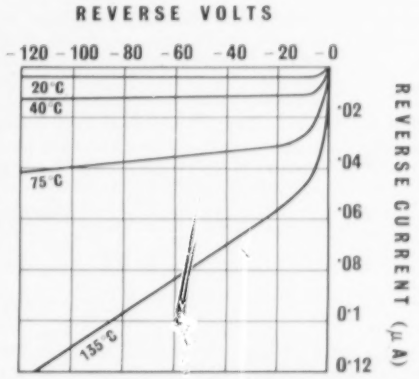

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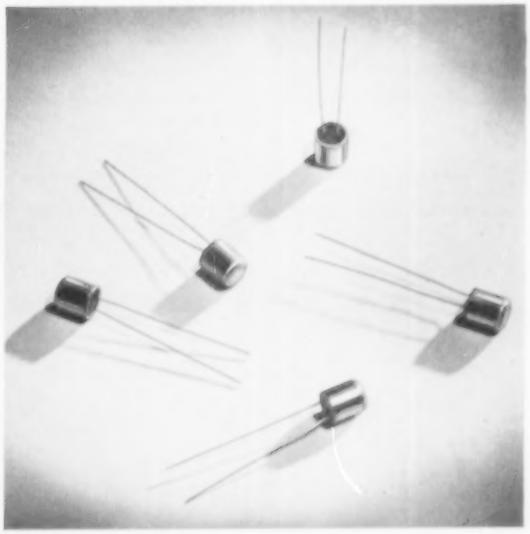

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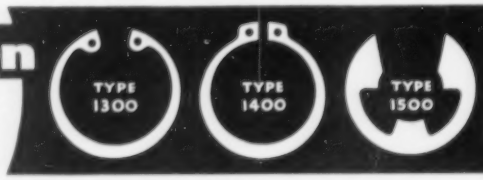

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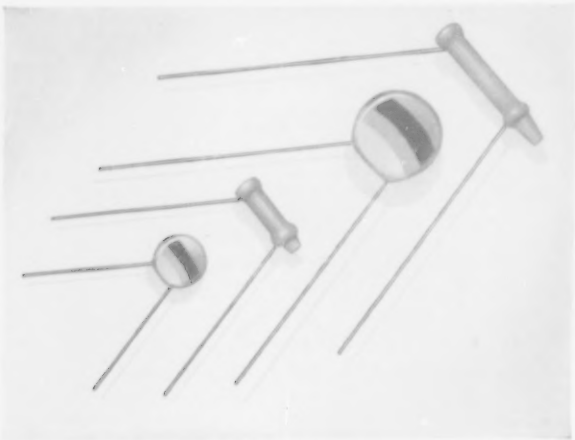

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Capacitance Range Picofarads	Length of Insulated Tube Millimetres
*Type P100. Temperature Coefficient (+100 ± 100) × 10^{-6} pF/pF/°C	
1.5 to 7	11
7.1 to 11	13
11.1 to 16	16
16.1 to 26	21
26.1 to 33	26
33.1 to 40	31
40.1 to 50	36
50.1 to 60	41
*Type N33. Temperature Coefficient (−33 ± 60) × 10^{-6} pF/pF/°C	
5.0 to 27	11
27.1 to 45	13
45.1 to 69	16
69.1 to 100	21
100.1 to 140	26
140.1 to 180	31
180.1 to 220	36
220.1 to 250	41
*Type N750. Temperature Coefficient (−750 ± 250) × 10^{-6} pF/pF/°C	
10.0 to 80	11
80.1 to 110	13
110.1 to 180	16
180.1 to 240	21
240.1 to 350	26
350.1 to 450	31
450.1 to 550	36
550.1 to 650	41

*Capacitors with closer limits of Temperature Coefficient can be supplied by special arrangement.

HIGH-K TUBULAR
STANDARD CAPACITANCE RANGE

Capacitance Picofarads	Length of Insulated Tube Millimetres	
	TB1000	TB3000
470	11	—
680	11	—
800	11	11
1000	13	11
1500	16	11
2000	19	11
2200	19	11
3000	—	16
3300	—	16
4000	—	19
4700	—	19
5000	—	21

POWER CERAMICS

Ranges of Power Ceramics and Temperature Compensating types are also available.
Details on application.

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STANDARD CAPACITANCE RANGE

Capacitance Picofarads	Type Number	Diameter (over insulation) Millimetres
470	CD8K/2	9
680	CD8K/2	9
1000	CD9K/2	10.5
1500	CD9K/2	10.5
2200	CD11K/2	12
3300	CD12K/2	13
4700	CD14K/2	14.5

MATERIAL EMPLOYED

The temperature coefficient graph of the material is such that the capacitance increases with temperature from 20°C until it reaches a peak at approximately 50°C. With a further increase of temperature the capacitance falls, reaching its 20°C value again at approximately 80°C.
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